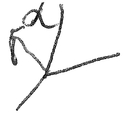
I-I Same / Some

$$u_{2x}^{\alpha} = \mathbb{I} \times (c\,\mathbb{I} + i\sigma_x s)$$

Chem Shift Rot

(eg) $c(\phi) = c(\frac{\phi}{2})$

$\sigma_x = 2I_x$



Start w/ matrix

$$= c\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} + is\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= c\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + is\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix} = \begin{pmatrix}c&0&is&0\\0&c&0&is\\is&0&c&0\\0&is&0&c\end{pmatrix} = u_{2x}^{\alpha}$$

$$\alpha_x = \frac{\phi_x}{2}$$

$$\sigma_x = 2I_x$$

$$e^{i\phi_x I_x} = e^{i\frac{\phi_x}{2}2I_x} = e^{i\alpha_x\sigma_x} = c(\alpha_x) + i\sigma_x s(\alpha_x)$$

$$u_{1x}^{-\alpha} = (c\,\mathbb{I} - i\sigma_x s)\otimes\mathbb{I} = c\,\mathbb{I} - is\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$= c\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} - is\,\underset{\begin{pmatrix}0&1\\1&0\end{pmatrix}}{\sigma_x}\otimes\underset{\begin{pmatrix}1&0\\0&1\end{pmatrix}}{1} =$$

$$u_{1x}^{-} = \begin{pmatrix}c&-is&0&0\\-i&c&0&0\\0&0&c&-is\\0&0&-is&c\end{pmatrix}$$

$$u_{1x} = \begin{pmatrix}c&is&0&0\\is&c&0&0\\0&0&c&is\\0&0&is&c\end{pmatrix}$$

$$u_{2x}^{-} = \begin{pmatrix}c&0&-is&0\\0&c&0&-is\\-is&0&c&0\\0&-is&0&c\end{pmatrix}$$

$$u_{2x} = \begin{pmatrix}c&0&is&0\\0&c&0&is\\is&0&c&0\\0&is&0&c\end{pmatrix}$$

$U_y = C(\phi)\mathbb{I} + i\,\sigma_y\,S(\phi)$

$$\boxed{\frac{\alpha}{2} = \phi \quad \therefore\ \alpha = 90 \Rightarrow \phi = 45^\circ}$$

L ~~***~~ careful

$\phi = \frac{\alpha}{2}$ $\quad \alpha \Rightarrow$

$I_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\sigma_y = 2 I_y$

Chem shift
Rotate
U

$U_{1x} \Rightarrow iSi\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -S\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$\therefore U_{1y} = \begin{pmatrix} C & -S & 0 & 0 \\ S & C & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{pmatrix}$ $\qquad U^{-}_{1y} = U^{+*}_{1y} = \begin{pmatrix} C & S & 0 & 0 \\ -S & C & 0 & 0 \\ 0 & 0 & C & S \\ 0 & 0 & -S & C \end{pmatrix}$

$$U_{2y} = \mathbb{I} \otimes [C\mathbb{I} + iS\sigma_y] = C\mathbb{I} + iSi\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
$$= C\mathbb{I} \underbrace{- S\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}}$$

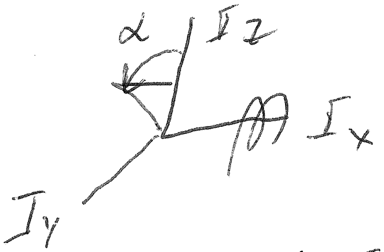
$C\mathbb{I} \qquad \Rightarrow -S\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

$\therefore U_{2y} = \begin{pmatrix} C & 0 & +S & 0 \\ 0 & C & 0 & +S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{pmatrix}$ $\qquad U^{-}_{2x} = U^{+*}_{2x} = \begin{pmatrix} C & 0 & -S & 0 \\ 0 & C & 0 & -S \\ S & 0 & C & 0 \\ 0 & S & 0 & C \end{pmatrix}$

$$U_{2z} = \begin{array}{c} \underline{\text{Diag}} \\ c - iS \\ c - iS \\ c + iS \\ c + iS \end{array} \qquad U_{2\bar{z}} = \begin{array}{c} \underline{\text{Diag}} \\ c + iS \\ c + iS \\ c - iS \\ c - iS \end{array} \qquad \alpha = \frac{\delta\omega_2 t}{2}$$

$$U_{1z} = \begin{array}{c} c - iS \\ c + iS \\ c - iS \\ c + iS \end{array} \qquad U_{1\bar{z}} = \begin{array}{c} c + iS \\ c - iS \\ c + iS \\ c - iS \end{array} \qquad \alpha = \frac{\delta\omega_1 t}{2}$$

V-II)



α= Pulse angle.

*Describe by unitary Rotations

$\sigma_x = 2I_x$

$\frac{\alpha}{2} = \phi$

$\sigma_x^2 = 1$

$\sigma_x^3 = \sigma_x$

$$u^{i\alpha I_x} = u^{i(\frac{\alpha}{2})I_x 2} = u^{i\phi\sigma_x}$$

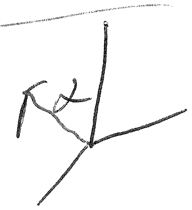
$$u^{i\phi\sigma_x} = 1 + \frac{i\phi\sigma_x}{1!} + \frac{(i\phi\sigma_x)^2}{2!} + \frac{(i\phi\sigma_x)^3}{3!} + \dots$$

$$= 1 - \frac{(i\phi)^2}{2!} + \frac{(i\phi)^4}{4!} - \frac{\phi^6}{6!} \Leftarrow C(\phi)^{\cancel{2}}$$

$$+ \frac{(i\phi\sigma_x)}{1!} - \frac{i\phi^3\sigma_x}{3!} + \frac{i\sigma_x\phi^5}{5!} \Leftarrow i\sigma_x S(\phi)$$

$\sigma_x = 2I_x$

$\phi = \frac{\alpha}{2}$



$$\therefore u^{i\alpha\sigma_x} = C(\phi) + i\sigma_x S(\phi)$$

$$u^{-i\alpha\sigma_x} = C(\phi) - i\sigma_x S(\phi)$$

$I_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$ $\mathbb{1} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$$u_x = C(\phi)\mathbb{1} + i\begin{pmatrix}0&1\\1&0\end{pmatrix}S(\phi)$$

$$u_x^- = C(\phi)\mathbb{1} - i\begin{pmatrix}0&1\\1&0\end{pmatrix}S(\phi)$$

$F_z = I_{1z} + I_{2z}$

*2 spins ∴ expand basis → 4×4

Note need 2 u_x for I_{1z} & I_{2z}

eg) AX u_{1x} & u_{2x}

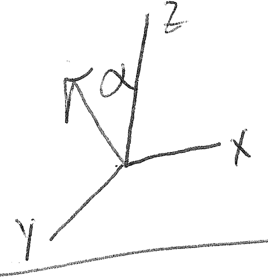
u_{1x}^{-1} & u_{2x}^{-1}

$$u_{1x} = C\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} + iS\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$= C\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} + iS\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$=$$

$$= \begin{pmatrix}C&iS&0&0\\iS&C&0&0\\0&0&C&iS\\0&0&iS&C\end{pmatrix} = u_{1x}$$



$\frac{\alpha}{2} = \frac{\phi}{2}$ $C(45) = \frac{1}{\sqrt{2}}$ $S(45) = \frac{1}{\sqrt{2}}$

$$U_{1y}(45)\, I_{1z}\, U_{1y}^{-}(45) = U_{1y}\left(\frac{\sigma_{1z}}{2}\right)U_{1y}^{-}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} \frac{1}{\sqrt{2}}$$

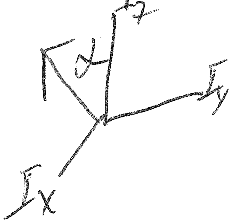
$$= \frac{1}{4}\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & -2 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$= \frac{\sigma_{1x}}{2} = -I_{1x}$$

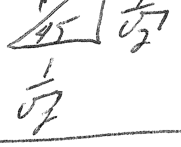
~~Ex~~ $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_{1x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

$\sigma_x = 2I_{1x}$

$I_{1x} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$



$\alpha = 90 \therefore \phi = 45 \quad c(45) = s(45) = \frac{1}{\sqrt{2}}$



$\sigma_{2z} = 2 I_{2z} \quad I_{2z} = \frac{\sigma_{2z}}{2}$

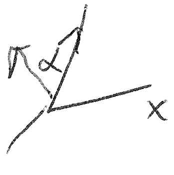
$$U_{2y}(45) I_{2z} U_{2y}^{-1}(45) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \Rightarrow \frac{1}{4} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & -2 \\ -2 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{pmatrix}$$

$$-I_{2x} \quad = -\frac{1}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = -\frac{\sigma_x}{2}$$

$\boxed{\#1}$

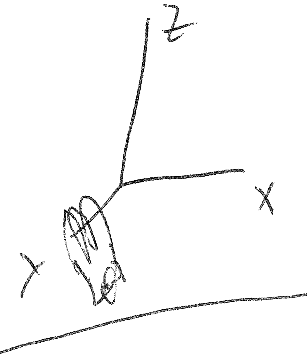
$$F_z \xrightarrow{\left(\frac{\pi}{2}\right)_y} U_{1z} I_{1z} U_{1z} + U_{2z} I_{2z} U_{2z}$$



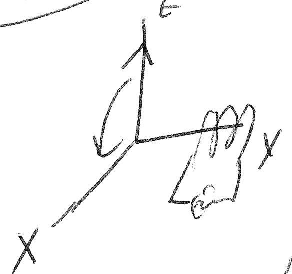
$$= -I_{1x} - I_{2x}$$

Sign diff here ~~I_{as}~~

I use: $e^{i\alpha_x I_y} I_z e^{-i\alpha_x I_y}$



Others: $e^{-i\alpha I_y} I_z e^{i\alpha I_y}$



II-1 B) $F_x \xrightarrow{\Delta\omega_1 I_{1z} t} I_{1x}\, c(\Delta\omega_1 t) \cancel{I_{1z}} + I_{1y}\, s(\Delta\omega_1 t)$

$\Delta\omega_2 I_{2z} t \quad + I_{2x}\, c(\Delta\omega_2 t) + I_{2y}\, s(\Delta\omega_2 t)$

$F_x = I_{1x} + I_{2x} = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}0&1\\1&0\end{pmatrix}$

$$F_x = \frac{1}{2}\left\{ \overset{I_{1x}}{\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}} + \overset{I_{2x}}{\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}} \right\} = \begin{pmatrix}0&1&1&0\\1&0&0&1\\1&0&0&0\\0&1&1&0\end{pmatrix}\frac{1}{2}$$

$$e^{-i\Delta\omega_1 t I_{1z}} = e^{-i\left(\frac{\Delta\omega_1 t}{2}\right)2 I_{1z}} = e^{-i\alpha\sigma_z} = \mathbb{1}\, c(\alpha) - i\sigma_{1z}\, s(\alpha)$$

$$e^{i\Delta\omega_1 t I_{1z}} = \mathbb{1}\, c(\alpha) + i\sigma_{1z}\, s(\alpha)$$

$$U_{1z}^{\alpha} = c\mathbb{1} - i\begin{pmatrix}1&0\\0&-1\end{pmatrix} s \otimes \begin{pmatrix}1&0\\0&1\end{pmatrix} = -is\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} + \begin{pmatrix}1&&0\\&1&\\0&&1\end{pmatrix}^{c}$$

$$U_{1z} = \overset{\text{Diag}}{\begin{pmatrix}c \mp is\\c+is\\c-is\\c+is\end{pmatrix}} \qquad U_{1z}^{-} = \overset{\text{Diag}}{\begin{pmatrix}c+is\\c-is\\c+is\\c-is\end{pmatrix}} \qquad \alpha = \frac{\Delta\omega_1 \cancel{t}}{2}$$

$$e^{-i\Delta\omega_2 I_2 t} = e^{-i\left(\frac{\Delta\omega_2 t}{2}\right)2 I_{2z}} = e^{-i\alpha\sigma_z} = c(\alpha) - i\sigma_z\, s(\alpha)$$

$$U_{2z}^{2} = e$$

$$\sigma_{2z} = \begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}(-is) + c\begin{pmatrix}1&&0\\&1&\\0&&1\end{pmatrix}$$

$$U_{1z} I_{1x} U_{1z}^{-} = \frac{1}{2} U_{1z}\, \sigma_{1x}\, U_{1z}^{-1}$$

Let $A = c + is$

$B = c - is$

$$= \frac{1}{2}\begin{pmatrix} c-is \\ c+is \\ c-is \\ c+is \end{pmatrix}\begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix} \cancel{2} \begin{pmatrix} c+is \\ c-is \\ c+is \\ c-is \end{pmatrix}$$

$$\begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix}\begin{pmatrix} A&0&0&0\\ 0&B&0&0\\ 0&0&A&0\\ 0&0&0&B \end{pmatrix} = \begin{pmatrix} 0&B&0&0\\ A&0&0&0\\ 0&0&0&B\\ 0&0&A&0 \end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix} B&0&0&0\\ 0&A&0&0\\ 0&0&B&0\\ 0&0&0&A \end{pmatrix}\begin{pmatrix} 0&B&0&0\\ A&0&0&0\\ 0&0&0&B\\ 0&0&A&0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0&B^2&0&0\\ A^2&0&0&0\\ 0&0&0&B^2\\ 0&0&A^2&0 \end{pmatrix}$$

$$A^2 = (c+is)(c+is) = c^2 + 2ics - s^2$$

$$B^2 = (c-is)(c-is) = c^2 - 2ics - s^2 \qquad = (c^2 - s^2) = c(2\alpha)$$

$$2ics = \overset{+2i}{\frac{2i\, s(2\alpha)}{2}}$$

$$= i\, s(2\alpha)$$

$$\therefore\ A^2 = c(2\alpha) + i\, s(2\alpha)$$

$$B^2 = c(2\alpha) - i\, s(2\alpha)$$

$$\boxed{\alpha = \frac{\Delta\omega_1 t}{2}}$$

$$\frac{1}{2}\left[c(\Delta\omega_1 t)\overset{\sigma_{1x}}{\begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&1\\ 0&0&1&0 \end{pmatrix}} + i\, s(\Delta\omega_1 t)\overset{\sigma_{1x}}{\begin{pmatrix} 0&-1&0&0\\ 1&0&0&0\\ 0&0&0&-1\\ 0&0&1&0 \end{pmatrix}}\right)$$

$$\frac{\sigma_{Z\cancel{2}}}{2} = I_{1z}$$

$$\boxed{I_{1x}\, c(\Delta\omega_1 t) + i\, I_{1y}\, s(\Delta\omega_1 t) = U_{1z} I_{1x} U_{1z}^{-1}}$$

$$U_{2z} I_{2x} U_{2z}^{-} = \frac{1}{2} U_{2z} \sigma_{2x} U_{2z}^{-}$$

$$\sigma_{2x} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} c - is \\ c - is \\ c + is \\ c + is \end{pmatrix} \begin{pmatrix} \sigma_{2x} \end{pmatrix} \begin{pmatrix} c + is \\ c + is \\ c - is \\ c - is \end{pmatrix}$$

Let $A = c + is$

$B = c - is$

$$= \frac{1}{2} \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \end{pmatrix} = \frac{1}{2} \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \begin{pmatrix} 0 & 0 & B & 0 \\ 0 & 0 & 0 & B \\ A & 0 & 0 & 0 \\ 0 & A & 0 & 0 \end{pmatrix}$$

$$A^2 = (c + is)(c + is) = c(2\alpha) + is(2\alpha) = \frac{1}{2} \begin{pmatrix} 0 & 0 & B^2 & 0 \\ 0 & 0 & 0 & B^2 \\ A^2 & 0 & 0 & 0 \\ 0 & A^2 & 0 & 0 \end{pmatrix}$$

$$B^2 = (c - is)(c - is) = c(2\alpha) - is(2\alpha)$$

$$\frac{1}{2} \Bigg(\quad \frac{1}{2} \Bigg[c(2\alpha) \overset{\sigma_{2x}}{\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}} + is(2\alpha) \overset{\sigma_{2y}/i}{\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}}$$

$$\boxed{\begin{aligned} &\therefore U_{2z} I_{2x} U_{2z}^{-} = I_{2x} c(\omega_2 t) + I_{2y} s(\omega_2 t) \\ &\& \, U_{1z} I_{1x} U_{1z}^{-} = I_{1x} c(\omega_1 t) + I_{1y} s(\omega_1 t) \end{aligned}}$$

$$\sigma_{2y} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_{2y} = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

$$\nearrow$$

F_x

$\cancel{V}$-1c) $H_J = J I_{1z} I_{2z} \xrightarrow{J I_{1z} I_{2z} t} H_J(t) \quad e^{-i\Omega t} H_J e^{i\omega_1 t}$

$$U_{1z} = e^{iJ I_{1z} I_{2z} t} = e^{i\left(\frac{J I_{2z}}{2}\right) t) I_{1z}} = e^{i\alpha \sigma_{1z}}$$

$$\boxed{\alpha = \frac{J I_{2z} t}{2}}$$

$$U_{1z} = C(\alpha) + i\sigma_{1z} S(\alpha) \qquad \sigma_{1z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} iS(\alpha)$$

$$\therefore U_{1z} = \begin{array}{c} \underline{\text{Diag}} \\ C + iS \\ C - iS \\ C + iS \\ C - iS \end{array} \qquad U_{1\bar{z}} = \begin{array}{c} \underline{\text{Diag}} \\ C - iS \\ C + iS \\ C - iS \\ C + iS \end{array} = U_{1z}^{+*}$$

Just like Prob B) but here $\Delta\omega_1 t = J I_{2z} t$

Let $A = C + iS$ & $B = A - iS$

$$U_{1\bar{z}} I_{1x} U_{1z} = \frac{1}{2} \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \overset{\sigma_{1x}}{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}} \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & A \end{pmatrix} \begin{pmatrix} 0 & B & 0 & 0 \\ A & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & A & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & B^2 & 0 & 0 \\ A^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & B^2 \\ 0 & 0 & A^2 & 0 \end{pmatrix}$$

$$\alpha = \frac{J I_z t}{2}$$

$$A^2 = (C + iS)(C + iS) = C(2\alpha) + iS(2\alpha)$$

$$B^2 = C(2\alpha) - iS(2\alpha)$$

$c(2\alpha)$

$= c\left(\frac{J I_{2z} t}{2}\right)$

$c(\phi) =$

$= c(J I_{2z} t) \nearrow$

$\alpha = J I_z t$

$\phi = \frac{J \sigma_z t}{2}$

$\sigma_z = 2 I_z$

$\sigma_{2z}^2 = 1$

$\sigma_{2z}^3 = \sigma_z$

$$c(\phi) = 1 - \frac{(\phi)^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} = c\left(\frac{Jt}{2}\right)$$

$$s(\phi) = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} \qquad = \sigma$$

$$= \frac{\frac{J\sigma_z}{2}}{1!} - \frac{\left(\frac{J\sigma_z}{2}\right)^3}{3!} + \frac{\left(\frac{J\sigma_z}{2}\right)^5}{5!} = \sigma_z \left[\frac{\left(\frac{Jt}{2}\right)}{1!} - \frac{\left(\frac{Jt}{2}\right)^3}{3!} + \frac{\left(\frac{Jt}{2}\right)^5}{5!} + \ldots \right.$$

$$= \sigma_{2z}\, s\left(\frac{Jt}{2}\right)$$

$$A^2 = c(2\alpha) + i\, s(2\alpha) = c\left(\frac{Jt}{2}\right) + i\, \sigma_{2z}\, s\left(\frac{Jt}{2}\right)$$

$$+ 2i\, I_{2z}\, s\left(\frac{Jt}{2}\right)$$

$$= c\left(\frac{Jt}{2}\right)$$

$$B^2 = c(2\alpha) - i\, s(2\alpha) = c\left(\frac{Jt}{2}\right) - i\, \sigma_{2z}\, s\left(\frac{Jt}{2}\right)$$

$$= c\left(\frac{Jt}{2}\right) - 2i\, I_{2z}\, s\left(\frac{Jt}{2}\right)$$

$\alpha = \frac{J I_{2z} t}{2}$

$\sigma_{2z}^2 = 1$

$\sigma_{2z}^3 = \sigma_{2z}$

$\sigma_{2z} = 2 I_{2z}$

Back to $U^-_{1z} I_{1x} U_{1z}$

$$A^2 = C\left(\frac{Jt}{2}\right) + i\,2I_{2z}\,S\left(\frac{Jt}{2}\right)$$

$$B^2 = C\left(\frac{Jt}{2}\right) - i\,2I_{2z}\,S\left(\frac{Jt}{2}\right)$$

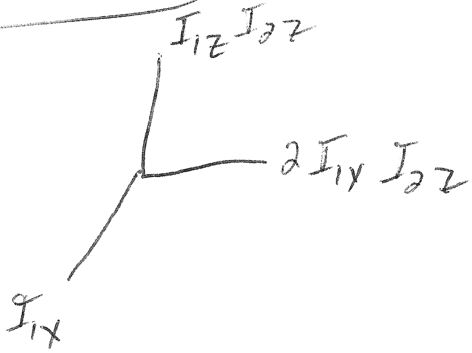
$$= \frac{1}{2}\begin{pmatrix} 0 & B^2 & 0 & 0 \\ A^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & B^2 \\ 0 & 0 & A^2 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\left[C\left(\frac{Jt}{2}\right)\overset{\sigma_{1x}}{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}} + 2I_{2z}\,S\left(\frac{Jt}{2}\right)\overset{\sigma_{1y}}{\begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}}\right.$$

$$\sigma_{1x} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \sigma_{1y} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$= \frac{1}{2}\left[2C\left(\frac{Jt}{2}\right)I_{1x} + 4\,I_{1y}I_{2z}\,S\left(\frac{Jt}{2}\right)\right]$$

$$\boxed{U^-_{1z} I_{1x} U_{1z}\left(\frac{J I_{2z} t}{2}\right) = I_{1x}\,C\left(\frac{Jt}{2}\right) + 2I_{1y}I_{2z}\,S\left(\frac{Jt}{2}\right)}$$



IV-1D)

$$2I_{1y}I_{2z} = 2\left(\frac{\sigma_{1y}}{2}\right)\left(\frac{\sigma_{2z}}{2}\right) = \frac{1}{2}\sigma_{1y}\sigma_{2z}$$

$$\sigma_{1y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}$$

$$\boxed{2I_{1y}I_{2z} \xrightarrow{JI_{1z}I_{2z}t} 2I_{1y}I_{2z}c\left(\frac{Jt}{2}\right) - I_x s\left(\frac{Jt}{2}\right)}$$

$$\sigma_{2z} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\frac{1}{2}\sigma_{1y}\sigma_{2z} = \frac{1}{2}i\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

$$\boxed{2I_{1y}I_{2z} = \frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}}$$

$$e^{-iJI_{1z}I_{2z}t} = \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & A \end{pmatrix}$$

$$A = c + \overset{\sigma_{2z}}{i}s\left(\frac{Jt}{2}\right)$$

$$B = c - \underset{\sigma_{2z}}{i}s\left(\frac{Jt}{2}\right)$$

$$A = c + is\left(\frac{Jt}{2}\right)\sigma_{2z}$$

$$B = c - is\left(\frac{Jt}{2}\right)\sigma_{2z}$$

$$\frac{i}{2}\begin{pmatrix} B & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & A \end{pmatrix}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & 0 & A & 0 \\ 0 & 0 & 0 & B \end{pmatrix} = \frac{i}{2}\begin{pmatrix} B & 0 & 0 & 0 \\ 0 & A & 0 & 0 \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & A \end{pmatrix}\begin{pmatrix} 0 & -B & 0 & 0 \\ A & 0 & 0 & 0 \\ 0 & 0 & 0 & B \\ 0 & 0 & -A & 0 \end{pmatrix}$$

$$= \frac{i}{2}\begin{pmatrix} 0 & -B^2 & 0 & 0 \\ A^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & B^2 \\ 0 & 0 & -A^2 & 0 \end{pmatrix}$$

$$A^2 = (c+is)(c+is) = c^2 s^2 + 2ics$$

$$= c(2\alpha) + is(2\alpha)$$

$$B^2 = c(2\alpha) - is(2\alpha)$$

$B^2 =$

~~$c\left(\frac{Jt}{2}\right)\left[\frac{i}{2}\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}\right] - \frac{i}{2}s\left(\frac{Jt}{2}\right)\begin{pmatrix} 0 & + \end{pmatrix}$~~

WRITE OUT IN SINGL MATRIX

$$\frac{i}{2}\begin{bmatrix} 0 & -[c - i\sigma_{2z} s] & 0 & 0 \\ c + i s \sigma_{2z} & 0 & 0 & 0 \\ 0 & 0 & 0 & c - i\sigma_{2z} s \\ 0 & 0 & -[c + i s \sigma_{2z}] & 0 \end{bmatrix}$$

$$A^2 = (c + i s \sigma_{2z})(c + i s \sigma_{2z}) = c^2 + i^2 s^2 \sigma_{2z}^2 + 2 i c s \sigma_{2z}$$

$$= c^2 - s^2 + 2 i c s \sigma_{2z}$$

$$= c(2\alpha) + i \sigma_{2z} s(2\alpha)$$

$$\alpha = \frac{Jt}{4}$$

$$\boxed{\begin{aligned} A^2 &= c\left(\tfrac{Jt}{2}\right) + i\sigma_{2z} s\left(\tfrac{Jt}{2}\right) \\ B^2 &= c\left(\tfrac{Jt}{2}\right) - i\sigma_{2z} s\left(\tfrac{Jt}{2}\right) \end{aligned}}$$

$$e^{+iJ I_{1z} I_{2z} t} = e^{i\left(\frac{J I_{2z} t}{2}\right) I_{1z} * 2} = e^{i\left(\frac{J I_{2z} t}{2}\right)\sigma_{1z}}$$

$$= e^{i\left(\frac{J I_{2z} t}{4}\right)\sigma_{1z}} = e^{i\left(\frac{J \sigma_{2z} t}{4}\right)\sigma_{1z}}$$

$$= e^{i\alpha\sigma_{1z}}$$

$$\alpha = \frac{J\sigma_{2z} t}{4}$$

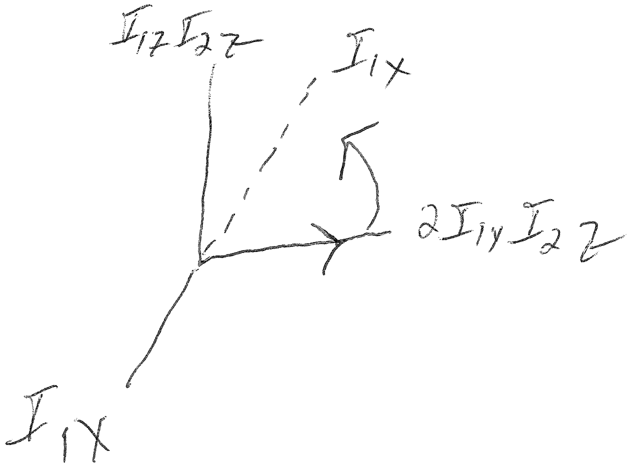
$$= \underbrace{\frac{i}{2} c \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}}_{2 I_{1y} I_{2z} c(2\alpha)} + i\sigma_{2z} s \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \frac{i}{2}$$

$$iS \overset{\sigma_{2z}}{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \frac{i}{2} = iS \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \frac{i}{2}$$

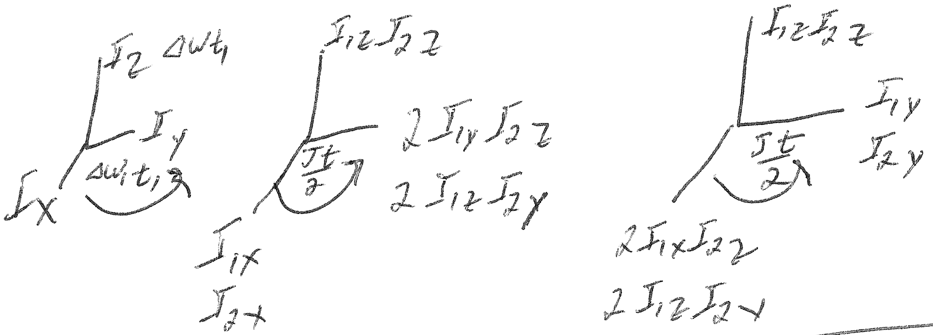
$$= -\frac{1}{2} S(2\alpha) \overset{\sigma_{1x}}{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}} = -I_{1x} S(2\alpha)$$

$$\alpha = \frac{Jt}{4}$$

$$\therefore \boxed{2I_{1y}I_{2z} \xrightarrow{JI_{1z}I_{2z}t} 2I_{1y}I_{2z}\, C\left(\frac{Jt}{2}\right) - I_{1x}\, S\left(\frac{Jt}{2}\right)}$$



V-2)



$$I_{1z} \xrightarrow{JI_{1z}I_{2z}t} I_{1z}$$

$$I_{1y} \xrightarrow{JI_{1z}I_{2z}t} I_{1y}\,C\left(\frac{Jt}{2}\right) - 2I_{1x}I_{2z}\,S(Jt/2)$$

$$2I_{1y}I_{2z} \xrightarrow{JI_{1z}I_{2z}t} 2I_{1y}I_{2z}\,C\left(\frac{Jt}{2}\right) - I_{1x}\,S\left(\frac{Jt}{2}\right)$$

$$2I_{1x}I_{2z} \xrightarrow{JI_{1z}I_{2z}t} 2I_{1x}I_{2z}\,C\left(\frac{Jt}{2}\right) + I_{1y}\,S\left(\frac{Jt}{2}\right)$$

$$I_{1x} \xrightarrow{JI_{1z}I_{2z}t} I_{1x}\,C\left(\frac{Jt}{2}\right) + 2I_{1y}I_{2z}\,S\left(\frac{Jt}{2}\right)$$

$$\xrightarrow{\Delta w_1 I_{1z} t}$$

$$[\cancel{I_{1x}}(\,[I_{1x}\,C(\Delta w_1 t) + I_{1y}\,S(\Delta w_1 t)]\,C\left(\frac{Jt}{2}\right)$$

$$2I_{1y}I_{2z} \xrightarrow{\Delta w_1 t I_{1z}} 2\,[I_{1y}\,C(\Delta w_1 t) - I_{1x}\,S(\Delta w_1 t)]\,I_{2z}\,S\left(\frac{Jt}{2}\right)$$

$$I_{1x} \xrightarrow{\Delta w_1 t I_{1z}} I_{1x}\,C(\Delta w_1 t) \xrightarrow{JI_{1z}I_{2z}t} C(\Delta w_1 t)\left[I_{1x}C\left(\frac{Jt}{2}\right) + 2I_{1y}I_{2z}\,S\left(\frac{Jt}{2}\right)\right]$$

$$+\ I_{1y}\,S(\Delta w_1 t) \xrightarrow{JI_{1z}I_{2z}s} S(\Delta w_1 t)\left[I_{1y}\,C\left(\frac{Jt}{2}\right) - 2I_{1x}I_{2z}\,S\left(\frac{Jt}{2}\right)\right]$$

$[CS, J] = 0$ ∴ order independant

V-3	$\begin{matrix}1&0\\0&1\end{matrix}$ $\mathbb{1}$	$\left(\frac{1}{2}\right)$ $\begin{matrix}0&1\\1&0\end{matrix}$ S_x	$\left(\frac{i}{2}\right)$ $\begin{matrix}0&-1\\1&0\end{matrix}$ S_y	$\left(\frac{1}{2}\right)$ $\begin{matrix}1&0\\0&-1\end{matrix}$ S_z
$\begin{matrix}1&0\\0&1\end{matrix}$ $\mathbb{1}$	$\boxed{\mathbb{1}}$ $\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}$	$\boxed{S_x}$ $\begin{matrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{matrix}$ $\left(\frac{1}{2}\right)$	$\boxed{S_y}$ $\begin{matrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{matrix}$ $\left(\frac{i}{2}\right)$	$\boxed{S_z}$ $\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{matrix}$ $\left(\frac{1}{2}\right)$
$\left(\frac{1}{2}\right)$ $\begin{matrix}0&1\\1&0\end{matrix}$ I_x	$\boxed{I_x}$ $\begin{matrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&0&0\end{matrix}$ $\left(\frac{1}{2}\right)$	$\boxed{I_xS_x}$ $\begin{matrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{matrix}$ $\left(\frac{1}{4}\right)$	$\boxed{I_xS_y}$ $\begin{matrix}0&0&0&-1\\0&0&-1&0\\0&1&0&0\\1&0&0&0\end{matrix}$ $\left(\frac{i}{4}\right)$	$\boxed{I_xS_z}$ $\begin{matrix}0&1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&-1&0\end{matrix}$ $\left(\frac{1}{4}\right)$
$\left(\frac{i}{2}\right)$ $\begin{matrix}0&-1\\1&0\end{matrix}$ I_y	$\boxed{I_y}$ $\begin{matrix}0&-1&0&0\\1&0&0&0\\0&0&0&-1\\0&0&1&0\end{matrix}$ $\left(\frac{i}{2}\right)$	$\boxed{I_yS_x}$ $\begin{matrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{matrix}$ $\left(\frac{i}{4}\right)$	$\boxed{I_yS_y}$ $\begin{matrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{matrix}$ $\left(\frac{-1}{4}\right)$	$\boxed{I_yS_z}$ $\begin{matrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{matrix}$ $\left(\frac{i}{4}\right)$
$\left(\frac{1}{2}\right)$ $\begin{matrix}1&0\\0&-1\end{matrix}$ I_z	$\boxed{I_z}$ $\begin{matrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{matrix}$ $\left(\frac{1}{2}\right)$	$\boxed{I_zS_x}$ $\begin{matrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{matrix}$ $\left(\frac{1}{4}\right)$	$\boxed{I_zS_y}$ $\begin{matrix}0&0&-1&0\\0&0&0&1\\1&0&0&0\\0&-1&0&0\end{matrix}$ $\left(\frac{i}{4}\right)$	$\boxed{I_zS_z}$ $\begin{matrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{matrix}$ $\left(\frac{1}{4}\right)$

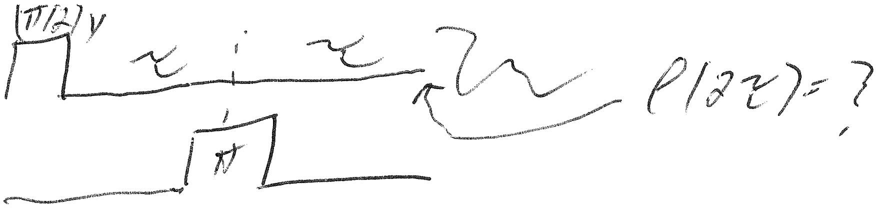
V-4)

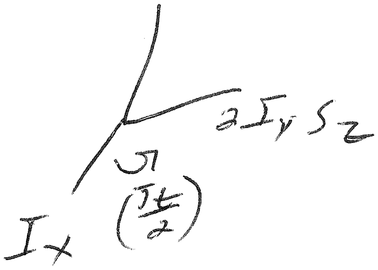
1	S_x	S_y	S_z	$1M_x$	S_xM_x	S_yM_x	S_zM_x
I_x	I_xS_x	I_xS_y	I_xS_z	I_xM_x	$I_xS_xM_x$	$I_xS_yM_x$	$I_xS_zM_x$
I_y	I_yS_x	I_yS_y	I_yS_z	I_yM_x	$I_yS_xM_x$	$I_yS_yM_x$	$I_yS_zM_x$
I_z	I_zS_x	I_zS_y	I_zS_z	I_zM_x	$I_zS_xM_x$	$I_zS_yM_x$	$I_zS_zM_x$
$1M_y$	S_xM_y	S_yM_y	S_zM_y	$1M_z$	S_xM_z	S_yM_z	S_zM_z
I_xM_y	$I_xS_yM_y$	$I_xS_yM_y$	$I_xS_zM_y$	I_xM_z	$I_xS_xM_z$	$I_xS_yM_z$	$I_xS_zM_z$
I_yM_y	$I_yS_xM_y$	$I_yS_yM_y$	$I_yS_zM_y$	I_yM_z	$I_yS_xM_z$	$I_yS_yM_z$	$I_yS_zM_z$
I_zM_y	$I_zS_xM_y$	$I_zS_yM_y$	$I_zS_zM_y$	I_zM_z	$I_zS_xM_z$	$I_zS_yM_z$	$I_zS_zM_z$

$$\begin{array}{lllll} \mathbb{1} & 1 & \mathbb{1} & S_x & S_y \quad S_z \\ I_x & & I_x & I_xS_y & \\ I_y & & I_y & & \\ I_z & & I_z & & \end{array}$$

$$\begin{pmatrix} \mathbb{1} & S_x & S_y & S_z \\ I_x & I_xS_x & I_xS_y & I_xS_z \\ I_y & I_yS_x & I_yS_y & I_yS_z \\ I_z & I_zS_x & I_zS_y & I_zS_z \end{pmatrix} \otimes \begin{pmatrix} \mathbb{1} & M_x \\ M_y & M_z \end{pmatrix}$$

II-5)





$$I_x \xrightarrow[\tau]{J} c I_x + 2I_yS_z s \xrightarrow[\pi_S]{} -2I_yS_z s$$

$$c\left[I_x c + 2I_yS_z s\right) = c^2 I_x + cs\,\underline{2I_yS_z}$$

$$-s\left[2I_yS_z c - I_x s\right) = -cs\,\cancel{2I_yS_z} + s^2 I_x$$

$$I_x(c^2+s^2) = I_x$$

$[H_{cs}, H_S] = 0$ ∴ order doesn't matter

Now H_{cs}

$$\boxed{I_x \xrightarrow[2\tau]{-\omega_I I_z} I_x c(\omega_I 2\tau) + I_y s(\omega_I 2\tau}$$

$(\rho(2\tau))$

① IV-6)

$$\boxed{H_{IS} = -\omega_S S_z - \omega_I I_z + J I_z S_z + \omega_2 I_x}$$

IS | WORKS
- benchmarked against shaka review

$$S_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{2} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\left(\frac{-\omega_S}{2}\right) \qquad \text{Diag} = \begin{pmatrix} -\omega_S \\ -\omega_S \\ +\omega_S \\ +\omega_S \end{pmatrix}\frac{1}{2}$$

$$I_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\left(\frac{-\omega_I}{2}\right) \qquad \text{Diag} = \begin{pmatrix} -\omega_I \\ \omega_I \\ -\omega_I \\ +\omega_I \end{pmatrix}\frac{1}{2}$$

$$J I_z S_z = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\left(\frac{J}{4}\right) = \qquad \text{Diag} = \begin{pmatrix} J/4 \\ -J/4 \\ -J/4 \\ J/4 \end{pmatrix}$$

$$I_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \frac{\omega_2}{2} & 0 & 0 \\ \frac{\omega_2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega_2/2 \\ 0 & 0 & \omega_2/2 & 0 \end{pmatrix}$$

Let $A = \frac{-\omega_S}{2} - \frac{\omega_I}{2} + \frac{J}{4}$

(Sum diag parts)

$B = -\frac{\omega_S}{2} + \frac{\omega_I}{2} - \frac{J}{4}$

$C = \frac{\omega_S}{2} - \frac{\omega_I}{2} - \frac{J}{4}$

$D = \frac{\omega_S}{2} + \frac{\omega_I}{2} + \frac{J}{4}$

$$H_T = \begin{pmatrix} A & \frac{\omega_2}{2} & 0 & 0 \\ \frac{\omega_2}{2} & B & 0 & 0 \\ 0 & 0 & C & \omega_2/2 \\ 0 & 0 & \frac{\omega_2}{2} & D \end{pmatrix}$$

③ $\det \begin{vmatrix} G-\lambda & \frac{w_2}{2} \\ \frac{w_2}{2} & H-\lambda \end{vmatrix} = 0$

$G = -\frac{w_S}{2} - \frac{w_I}{2} + \frac{J}{4}$

$H = -\frac{w_S}{2} + \frac{w_I}{2} - \frac{J}{4}$

$(G-\lambda)(H-\lambda) - \frac{w_2^2}{4}$

$G(H-\lambda) - \lambda(H-\lambda) - \frac{w_2^2}{4} = 0$

$GH - \lambda G - \lambda H + \lambda^2 - \frac{w_2^2}{4} = 0$

$\lambda^2 - \lambda(G+H) + GH - \frac{w_2^2}{4} = 0$

$$\boxed{\frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \lambda^{+/-}}$$

$A = 1 \quad B = (-G-H)$

$C = GH - \frac{w_2^2}{4}$

$$= -(-G-H) \pm \sqrt{(-G-H)(-G-H) - 4(\overset{1}{\cancel{-G-H}})(GH - \frac{w_2^2}{4})} \Big/ 2$$

$$= G+H \pm \sqrt{-G(-G-H) - H(-G-H) - 4GH + w_2^2} \Big/ 2$$

$$= G+H \pm (G^2 + GH + GH + H^2 - 4GH + w_2^2)^{1/2} / 2$$

$$= G+H \pm [G^2 + H^2 \cancel{2} - 2GH + w_2^2]^{1/2} / 2$$

$G = -\frac{w_S}{2} - \frac{w_I}{2} + \frac{J}{4}$ → $-w_S = G+H$

$+H = -\frac{w_S}{2} + \frac{w_I}{2} - \frac{J}{4}$

$G^2 = \frac{1}{2}(-w_S - w_I + \frac{J}{2})\frac{1}{2}(-w_S - w_I + \frac{J}{2})$

$$= \frac{1}{4}\left(\begin{array}{ll} -w_S(-w_S - w_I + \frac{J}{2}) & = 1\, w_S^2 + w_I w_S - w_S J/2 \\ -w_I(-w_S - w_I + \frac{J}{2}) & + w_I w_S + w_I^2 - w_I J/2 \\ + \frac{J}{2}(-w_S - w_I + \frac{J}{2}) & - \frac{w_S J}{2} - \frac{w_I J}{2} + J^2/4 \end{array} \right.$$

④ $G^2 = \frac{w_S^2}{4} + \frac{2w_I w_S}{4} - \frac{w_S J}{4} + \frac{w_I^2}{4} - \frac{w_I J}{4} + \frac{J^2}{16}$

$$-2GH = \frac{-2}{4}\left[\left(-w_S - w_I + \frac{J}{2}\right)\left(-w_S + w_I - \frac{J}{2}\right)\right)$$

$$= \frac{-1}{2}\left[-w_S\left(-w_S + w_I - \frac{J}{2}\right) = w_S^2 - w_I w_S + w_S J/2 \right] \frac{-1}{2}$$

$$-w_I\left(-w_S + w_I - \frac{J}{2}\right) = w_I w_S - w_I^2 + w_I J/2$$

$$+\frac{J}{2}\left(-w_S + w_I - \frac{J}{2}\right) = \cancel{-w_S J/2} + \frac{w_I J}{2} - J^2/4$$

$$= \left(w_S^2 - w_I^2 + w_I J - J^2/4\right)\left(\frac{-1}{2}\right)$$

$$= \cancel{\frac{4}{2}}\left(-\frac{w_S^2}{2} + \frac{w_I^2}{2} - \frac{w_I J}{2} + \frac{J^2}{8} = -2GH\right.$$

$$H = \left(-w_S + w_I - \frac{J}{2}\right)\frac{1}{2}$$

$$H^2 = \frac{1}{4}\left[-w_S\left(-w_S + w_I - \frac{J}{2}\right) = w_S^2 - w_I w_S + w_S J/2 \right.$$

$$w_I\left(-w_S + w_I - \frac{J}{2}\right) = -w_I w_S + w_I^2 - w_I J/2$$

$$-\frac{J}{2}\left(-w_S + w_I - \frac{J}{2}\right) = +w_S J/2 - \frac{w_I J}{2} + J^2/4$$

$$= \frac{1}{4}\left[w_S^2 - 2w_I w_S + w_S J + w_I^2 - w_I J + \frac{J^2}{4}\right.$$

$$H^2 = \frac{w_S^2}{4} - \frac{w_I w_S}{2} + \frac{w_S J}{4} + \frac{w_I^2}{4} - \frac{w_I J}{4} + \frac{J^2}{16}$$

$$G^2 = \frac{w_S^2}{4} + \frac{w_I w_S}{2} - \frac{w_S J}{4} + \frac{w_I^2}{4} - \frac{w_I J}{4} + \frac{J^2}{16}$$

$$+H^2 = \frac{w_S^2}{4} - \frac{w_I w_S}{2} + \frac{w_S J}{4} + \frac{w_I^2}{4} - \frac{w_I J}{4} + \frac{J^2}{16}$$

$$G^2 + H^2 = \frac{w_S^2}{2} + \frac{w_I^2}{2} - \frac{w_I J}{2} + \frac{J^2}{8}$$

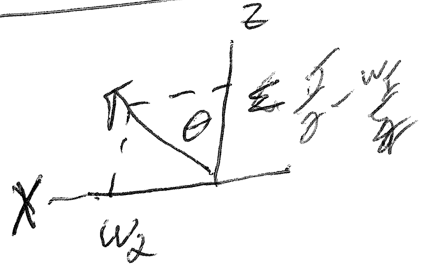
⑤ $G^2+H^2 = \frac{\omega_S^2}{2} + \frac{\omega_I^2}{2} - \frac{\omega_I J}{2} + \frac{J^2}{8}$

$-2GH - -\frac{\omega_S^2}{2} + \frac{\omega_I^2}{2} - \frac{\omega_I J}{2} + \frac{J^2}{8}$

$= \omega_I^2 - \omega_I J + \frac{J^2}{4} = G^2+H^2-2GH$

$\Sigma = -\frac{\omega_I}{2} - \frac{\omega_S}{2}$

$$\frac{-W_S \pm \sqrt{\omega_I^2 - \omega_I J + \frac{J^2}{4} + \omega_{\cancel{S}}^2}}{2}$$



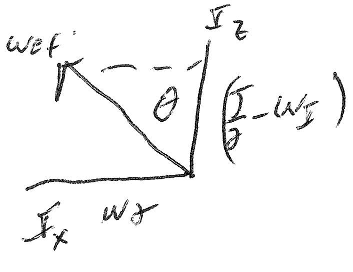
$$H_{AB} - \begin{pmatrix} -\frac{\cancel{\Delta}}{2}-\frac{J}{4} & \frac{J}{2} \\ \frac{J}{2} & \frac{\Delta}{2}-\frac{J}{4} \end{pmatrix} = H = -\frac{J}{4}\mathbb{1} \; \underline{-\Delta} I_z + J I_x$$

$$H_{Need} \begin{pmatrix} -\frac{\omega_S}{2}-\frac{\omega_I}{2}+\frac{J}{4} & \frac{\omega_2}{2} \\ \frac{\omega_2}{2} & -\frac{\omega_S}{2}+\frac{\omega_I}{2}-\frac{J}{4} \end{pmatrix} - I_x\omega_2 + I_z\frac{J}{2} + \mathbb{1}\left(-\frac{\omega_S}{2}\right) + \cancel{\mathbb{1}\left(\frac{\omega_I}{2}\right)}$$

$+ I_z(-\omega_I)$

$= I_x\omega_2 + I_z\left(\frac{J}{2}-\omega_I\right) + \mathbb{1}\left(-\frac{\omega_S}{2}\right)$

$\theta - \tan^{-1}\left(\frac{\omega_2}{\frac{J}{2}-\omega_I}\right)$



atan2(J, Δ)

$\omega_e = \left(\frac{J}{2}-\omega_I\right)\left(\frac{J}{2}-\omega_I\right) + \omega_2^2$ Wow!

$= \frac{J^2}{4} - \omega_I J + \omega_I^2 + \omega_2^2$

⑥ $\therefore \lambda^{\pm} = \frac{-W_S \pm W_{ef}}{2}$ $W_e = \sqrt{\frac{J^2}{4} - W_I J + W_I^2 + W_d^2}$

Find The ES's 1st Block

$$\lambda^{+} = -\frac{W_S + W_e}{2} \Rightarrow \begin{pmatrix} G - \frac{W_S + W_e}{2} & \frac{W_d}{2} \\ \frac{W_d}{2} & H - \frac{W_S + W_e}{2} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = 0$$

$$-\frac{W_S}{2} - \frac{W_I}{2} + \frac{J}{4} - \left(-\frac{W_S}{2} + \frac{W_e}{2}\right) = -\frac{W_I}{2} + \frac{J}{4} - \frac{W_e}{2}$$

$$-\frac{W_S}{2} + \frac{W_I}{2} - \frac{J}{4} - \left(-\frac{W_S}{2} + \frac{W_e}{2}\right) = -W_S + \overset{I}{\frac{W_I}{2}} - \frac{J}{4} - \frac{W_e}{2}$$

$$-\frac{W_I}{2} - \frac{J}{4} - \frac{W_e}{2} \quad \frac{W_d}{2}$$

~~$-\frac{W_I}{2} - \frac{W_S}{2}$~~

$$-\frac{W_I}{2} + \frac{J}{4} - \frac{W_e}{2} \quad \left(\frac{W_d}{2}\right)$$

$$\frac{W_d}{2} \quad -\frac{W_S}{2} + \frac{W_I}{2} - \frac{J}{4} - \frac{W_e}{2}$$

$$G = -\frac{W_S}{2} - \frac{W_I}{2} + \frac{J}{4}$$

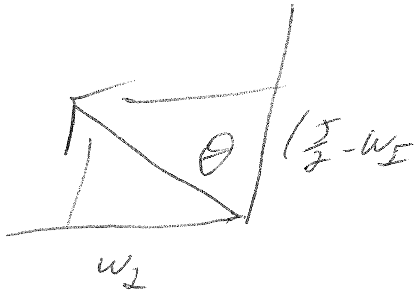
$$H = -\frac{W_S}{2} + \frac{W_I}{2} - \frac{J}{4}$$

$$G - \lambda = -\frac{W_S}{2} - \frac{W_I}{2} + \frac{J}{4} - \left(-\frac{W_S}{2} + \frac{W_e}{2}\right)$$
$$= -\frac{W_I}{2} + \frac{J}{4} - \frac{W_e}{2}$$

$$H - \lambda = -\frac{W_S}{2} + \frac{W_I}{2} - \frac{J}{4} - \left(-\frac{W_S}{2} + \frac{W_e}{2}\right)$$
$$= \frac{W_I}{2} - \frac{J}{4} - \frac{W_e}{2}$$

⑦

$$\begin{pmatrix} -\frac{w_I}{2}+\frac{J}{4}-\frac{w_e}{2} & \frac{w_2}{2} \\ \frac{w_2}{2} & \frac{w_I}{2}-\frac{J}{4}-\frac{w_e}{2} \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix}$$



$$\begin{pmatrix} -w_I+\frac{J}{2}-w_e & w_2 \\ w_2 & w_I-\frac{J}{2}-w_e \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix}$$

$$\begin{pmatrix} \frac{\frac{J}{2}-w_I}{w_e}-\frac{w_e}{w_e} & \frac{w_2}{w_e} \\ \frac{w_2}{w_e} & \frac{-(\frac{J}{2}-w_I)}{w_e}-\frac{w_e}{w_e} \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$$

$$\boxed{\begin{array}{l} \frac{J_2-w_I}{w_2}=t\theta \\ \hline \frac{w_2}{w_e}=s\theta \\ \hline \frac{J_2-w_I}{w_e}\ c\theta \\ \hline w_e = \frac{J^2}{4}-w_I J+w_I^2+w_2^2 \end{array}}$$

$$\begin{pmatrix} c\theta-1 & s\theta \\ s\theta & -(c\theta)-1 \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0 \qquad \begin{pmatrix} c_1(c-1) & c_2 s \\ \cancel{c_2(-c-1)}\ c_1 s & c_2(-1c-s) \end{pmatrix}$$

IDENTS

$$T\left(\frac{\theta}{2}\right)=\frac{s\theta}{1+c\theta}$$

$$T\left(\frac{\theta}{2}\right)=\frac{s\left(\frac{\theta}{2}\right)}{c\left(\frac{\theta}{2}\right)}$$

$$c_1(c-1)+c_2 s=0$$
$$c_1 s+c_2(-c-1)=0$$

$$c_1 s=c_2(c+1)$$

$$c_2=\frac{c_1 s}{c+1}=c_1\left(T\frac{\theta}{2}\right)$$

$$c_1^2+c_2^2=1$$

$$c_1^2+c_1^2T^2=1$$

$$c_1^2=\frac{1}{1+T^2}=\frac{1}{1+\frac{s^2\theta}{c^2\theta}}\overset{\times c^2}{=}\frac{c^2}{c^2+s^2}=c^2\frac{\theta}{2}$$

$$\therefore\ c_1=c\left(\frac{\theta}{2}\right) \qquad c_2=c_1T\left(\frac{\theta}{2}\right)=\frac{s\left(\frac{\theta}{2}\right)}{c\left(\frac{\theta}{2}\right)}c_1=s\frac{\theta}{2}$$

$$\boxed{\begin{array}{l} c_1=c\left(\frac{\theta}{2}\right) \\ c_2=s\left(\frac{\theta}{2}\right) \\ \hline \lambda^{+}=\frac{-w_I+w_e}{2} \end{array}}$$

⑧ $\lambda^{-} = -\frac{\omega_S}{2} - \frac{\omega_e}{2}$

$$G = -\frac{\omega_S}{2} - \frac{\omega_I}{2} + \frac{J}{4}$$

$$H = -\frac{\omega_S}{2} + \frac{\omega_I}{2} - \frac{J}{4}$$

$$\det\begin{vmatrix} G-\lambda & \frac{\omega_2}{2} \\ \frac{\omega_2}{2} & H-\lambda \end{vmatrix} = 0$$

$$G-\lambda = -\frac{\omega_S}{2} - \frac{\omega_I}{2} + \frac{J}{4} - \left(-\frac{\omega_S}{2} - \frac{\omega_e}{2}\right)$$

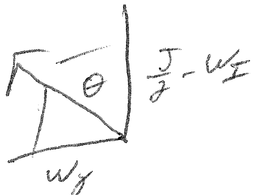
$$= \cancel{-\frac{\omega_S}{2}} - \frac{\omega_I}{2} + \frac{J}{4} + \cancel{\frac{\omega_S}{2}} + \frac{\omega_e}{2}$$

$$\boxed{G-\lambda = -\frac{\omega_I}{2} + \frac{J}{4} + \frac{\omega_e}{2}}$$

$$H-\lambda = -\frac{\omega_S}{2} + \frac{\omega_I}{2} - \frac{J}{4} - \left(-\frac{\omega_S}{2} - \frac{\omega_e}{2}\right) = \cancel{-\frac{\omega_S}{2}} + \frac{\omega_I}{2} - \frac{J}{4} + \cancel{\frac{\omega_S}{2}} + \frac{\omega_e}{2}$$

$$H-\lambda = \frac{\omega_I}{2} - \frac{J}{4} + \frac{\omega_e}{2}$$

$$\begin{pmatrix} -\frac{\omega_I}{2} + \frac{J}{4} + \frac{\omega_e}{2} & \frac{\omega_2}{2} \\ \frac{\omega_2}{2} & \frac{\omega_I}{2} - \frac{J}{4} + \frac{\omega_e}{2} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -\omega_I + \frac{J}{2} \cancel{+} \omega_e & \omega_2 \\ \omega_2 & \omega_I - \frac{J}{2} + \omega_e \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$$



$$\begin{pmatrix} \frac{\frac{J}{2} - \omega_I}{\omega_e} + \frac{\omega_e}{\omega_e} & \frac{\omega_2}{\omega_e} \\ \frac{\omega_2}{\omega_e} & \frac{-\left(\frac{J}{2} - \omega_I\right) + \omega_e}{\omega_e} \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$$

$$\boxed{\begin{aligned} T\theta &= \frac{\omega_2}{\frac{J}{2} - \omega_I} \\ c\theta &= \frac{\frac{J}{2} - \omega_I}{\omega_e} \\ s\theta &= \frac{\omega_2}{\omega_e} \end{aligned}}$$

$$\begin{pmatrix} c\theta + 1 & s\theta \\ s\theta & -c\theta + 1 \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$$

W

$$\boxed{\begin{aligned} T\frac{\theta}{2} &= \frac{s\theta}{1 + c\theta} \\ T\frac{\theta}{2} &= \frac{1 - c\theta}{s\theta} \end{aligned}}$$

$$c_1(c\theta + 1) + c_2 s\theta = 0$$

$$c_1 s\theta + c_2(-c\theta + 1) = 0$$

$$c_1 s\theta = \cancel{-c_2(c\theta - 1)} = -c_2(-c + 1) = \cancel{c_2(c - 1)}$$
$$= -c_2(1 - c)$$

$$\therefore c_1 = \frac{-c_2(1 - c)}{s} = -c_2 T\left(\frac{\theta}{2}\right)$$

⑨

$c_1 = -c_2\, T\left(\frac{\theta}{2}\right)$ $\qquad c_1^2 + c_2^2 = 1 \qquad c_2^2 + c_2^2\, T^2\left(\frac{\theta}{2}\right) = 1$

$$c_2^2 = \frac{1}{1+T^2\left(\frac{\theta}{2}\right)} = \frac{1}{1+\frac{S^2}{C^2}} = \frac{C^2}{1}$$

$c_2^2 = C^2\left(\frac{\theta}{2}\right) \qquad c_2 = C\left(\frac{\theta}{2}\right)$

$c_1 = -c_2\left(\frac{S(\theta/2)}{C(\theta/2)}\right) = -S\left(\frac{\theta}{2}\right)$

$$\boxed{\begin{array}{l} \lambda^{-} = -\frac{\omega_S}{2} - \frac{\omega_e}{2} \\ \hline c_1 = -S\left(\frac{\theta}{2}\right) \\ c_2 = C\left(\frac{\theta}{2}\right) \end{array}}$$

$\Leftarrow$ S

		EV	
Block I)	λ^{+}	$\underline{-\frac{\omega_S}{2} + \frac{\omega_e}{2}}$	$C\left(\frac{\theta}{2}\right)\vert\beta\alpha\rangle + S\left(\frac{\theta}{2}\right)\vert\alpha\beta\rangle$
	λ^{-}	$\underline{-\frac{\omega_S}{2} - \frac{\omega_e}{2}}$	$-S\left(\frac{\theta}{2}\right)\vert\beta\alpha\rangle + C\left(\frac{\theta}{2}\right)\vert\alpha\beta\rangle$

$= \vert 1\rangle$

$= \vert 2\rangle$

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⑩ Block IV

$$\det \begin{vmatrix} G-\lambda & \frac{w_x}{2} \\ \frac{w_x}{2} & H-\lambda \end{vmatrix} = 0$$

$$G = \frac{w_S}{2} - \frac{w_I}{2} - \frac{J}{4} \quad H = \frac{w_S}{2} + \frac{w_I}{2} + \frac{J}{4}$$

$$\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\Rightarrow G + H \pm \left(G^2 + H^2 - 2GH + w_x{}^2 \right)^{1/2} / 2$$

$$G = \frac{w_S}{2} - \frac{w_I}{2} - \frac{J}{4} \qquad \therefore G + H = w_S$$

$$H = \frac{w_S}{2} + \frac{w_I}{2} + \frac{J}{4}$$

$$G^2 = \frac{1}{4}\left(w_S - w_I - \frac{J}{2}\right)\left(w_S - w_I - \frac{J}{2}\right)$$

$$= \frac{1}{4}\left[\begin{array}{ll} w_S\left(w_S - w_I - \frac{J}{2}\right) & = w_S^2 - w_S w_I - w_S J/2 \\ -w_I\left(w_S - w_I - \frac{J}{2}\right) & = -w_S w_I + w_I^2 + w_I J/2 \\ -\frac{J}{2}\left(w_S - w_I - \frac{J}{2}\right) & = -\frac{w_S J}{2} + \frac{w_I J}{2} + \frac{J^2}{4} \end{array}\right. = \left(\begin{array}{l} w_S^2 - 2 w_S w_I - w_S J \\ + w_I^2 + w_I J + \frac{J^2}{4} \end{array}\right)\frac{1}{4}$$

$$-2GH = \frac{-2}{4}\left(\begin{array}{l} w_S\left(w_S + w_I + J/2\right) \\ -w_I\left(w_S + w_I + J/2\right) \\ -J/2\left(w_S + w_I + J/2\right) \end{array}\right) = \frac{-1}{2}\left(\begin{array}{l} w_S^2 + w_S w_I + \cancel{w_S J/2} \\ -w_S w_I - w_I^2 - w_I J/2 \\ -\cancel{\frac{w_S J}{2}} - \frac{w_I J}{2} - \frac{J^2}{4} \end{array}\right)$$

$$= \frac{-1}{2}\left[w_S^2 - w_I^2 - w_I J - \frac{J^2}{4} \right]$$

$$① \quad H^2 = \frac{1}{4}\left[\begin{array}{lcl} w_S\left(w_S + w_I + \frac{J}{2}\right) & = & w_S^2 + w_S w_I + w_S J/2 \\ w_I\left(w_S + w_I + \frac{J}{2}\right) & = & w_S w_I + w_I^2 + w_I J/2 \\ \frac{J}{2}\left(w_S + w_I + \frac{J}{2}\right) & = & w_S J/2 + \frac{w_I J}{2} + \frac{J^2}{4} \end{array}\right.$$

$$= \frac{1}{4}\left[w_S^2 + 2 w_S w_I + w_S J + w_I^2 + w_I J + \frac{J^2}{4} \right)$$

$$G^2 = \frac{w_S^2}{4} - \frac{w_S w_I}{2} - \frac{w_S J}{4} + \frac{w_I^2}{4} + \frac{w_I J}{4} + \frac{J^2}{16}$$

$$+H^2 = \frac{w_S^2}{4} + \frac{w_S w_I}{2} + \frac{w_S J}{4} + \frac{w_I^2}{4} + \frac{w_I J}{4} + \frac{J^2}{16}$$

$$G^2 + H^2 = \frac{w_S^2}{2} + \frac{w_I^2}{2} + \frac{w_I J}{2} + \frac{J^2}{8}$$

$$G^2 + H^2 = \frac{w_S^2}{2} + \frac{w_I^2}{2} + \frac{w_I J}{2} + \frac{J^2}{8}$$

$$-2GH = \frac{-w_S^2}{2} + \frac{w_I^2}{2} + \frac{w_I J}{2} + \frac{J^2}{8}$$

$$+w_x^2$$

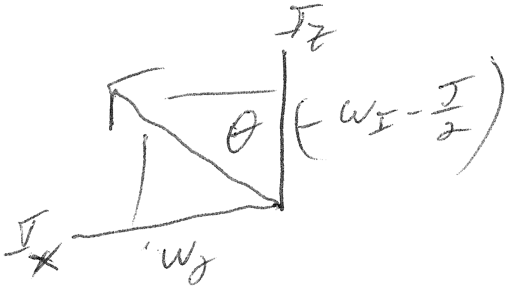
$$\rightarrow \sqrt{w_I^2 + w_I J + \frac{J^2}{4} + w_x^2}$$

$$\boxed{\lambda^{+}_{-} = \frac{w_S \pm \sqrt{w_I^2 + w_I J + \frac{J^2}{4} + w_x^2}}{2}}$$

b] $H_{B2} = \begin{pmatrix} C & w_2/2 \\ \frac{w_2}{2} & D \end{pmatrix} = \begin{pmatrix} \frac{w_S}{2} - \frac{w_I}{2} - \frac{J}{4} & \frac{w_2}{2} \\ \frac{w_2}{2} & \frac{w_S}{2} + \frac{w_I}{2} + \frac{J}{4} \end{pmatrix}$

$H_{B2} = \frac{w_S}{2}\mathbb{1} - I_z w_I - \frac{J}{2} I_z + I_x w_2 =$ Fictition $s = 1/2$

$$\boxed{H_{B2} = \frac{w_S}{2}\mathbb{1} + I_z\left(-w_I - \frac{J}{2}\right) + I_x w_2}$$



$T\theta = \frac{w_2}{-w_I - \frac{J}{2}}$

$w_e = \left(-w_I - \frac{J}{2}\right)\left(-w_I - \frac{J}{2}\right) + w_2^2 = -w_I\left(-w_I - \frac{J}{2}\right) - \frac{J}{2}\left(-w_I - \frac{J}{2}\right) + w_2^2$

$= w_I^2 + \frac{w_I J}{2} + \frac{w_I J}{2} + \frac{J^2}{4} + w_2^2$

$$\boxed{w_e = \sqrt{w_I^2 + w_I J + \frac{J^2}{4} + w_2^2}}$$

$T\theta = \frac{w_2}{-w_I - \frac{J}{2}}$ | $S\theta = \frac{w_2}{w_e}$ | $C\theta = \left(\frac{-w_I - J/2}{w_e}\right)$

Block II $\lambda^{+/-} = \frac{w_S \pm w_e}{2}$

$\begin{pmatrix} C - \lambda & \frac{w_2}{2} \\ \frac{w_2}{2} & D - \lambda \end{pmatrix} \begin{matrix} C_1 \\ C_2 \end{matrix} = 0$

NOW

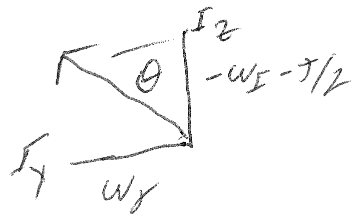
⑬ $\boxed{\lambda^+ = \frac{w_S}{2} + \frac{w_e}{2}} \Rightarrow \begin{pmatrix} C-\lambda & \frac{w_2}{2} \\ \frac{w_2}{2} & D-\lambda \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$

$\text{atan2} = \frac{Y}{X} = \frac{\text{Sin}}{\text{Cos}}$

$$C-\lambda = \frac{w_S}{2} - \frac{w_I}{2} - \frac{J}{4} - \left(\frac{w_S}{2} + \frac{w_e}{2}\right) = -\frac{w_I}{2} - \frac{J}{4} - \frac{w_e}{2}$$

$$D-\lambda = \frac{w_S}{2} + \frac{w_I}{2} + \frac{J}{4} - \left(\frac{w_S}{2} + \frac{w_e}{2}\right) = \frac{w_I}{2} + \frac{J}{4} - \frac{w_e}{2}$$

$$\begin{pmatrix} -\frac{w_I}{2} - \frac{J}{4} - \frac{w_e}{2} & \frac{w_2}{2} \\ \frac{w_2}{2} & \frac{w_I}{2} + \frac{J}{4} - \frac{w_e}{0} \end{pmatrix} = \begin{pmatrix} -w_I - \frac{J}{2} - w_e & w_2 \\ w_2 & w_I + \frac{J}{2} - w_e \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix}$$



$$= \begin{pmatrix} \frac{-w_I - J/2}{w_e} - \frac{w_e}{w_e} & \frac{w_2}{w_e} \\ \frac{w_2}{w_e} & \frac{w_I + \frac{J}{2}}{w_e} - \frac{w_e}{w_e} \end{pmatrix}$$

$$= \begin{pmatrix} C-1 & S \\ S & -C-1 \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix}$$

$c_1(C-1) + c_2 S = 0$

$c_1 S + c_2(-C-1) = 0$

$S c_1 = -c_2(-C-1) \nearrow \frac{S c_1}{C+1} = c_2$

$c_2 = c_1 T\left(\frac{\theta}{2}\right)$

$c_1^2 + c_1^2 T^2 = 1$

$c_1^2 = \frac{1}{1+T^2}$

✓ IDENTICAL TO 1ST BLOCK

$\therefore \quad c_1 = C\left(\frac{\theta}{2}\right)$

$c_2 = S\left(\frac{\theta}{2}\right)$

$\lambda^+ = \frac{w_S}{2} + \frac{w_e}{2}$

$c_1^2 = C^2\left(\frac{\theta}{2}\right)$

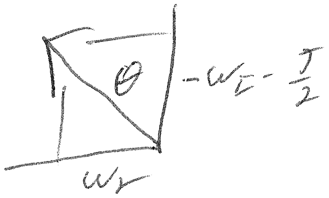
$c_1 = C\left(\frac{\theta}{2}\right)$

$c_2 = c_1 T$

$c_2 = C\left(\frac{S}{C}\right) = S\left(\frac{\theta}{2}\right)$

⑭ $\lambda^-_D = \frac{w_S}{2} - \frac{w_e}{2}$ $\quad \begin{pmatrix} C-\lambda & \frac{w_2}{2} \\ \frac{w_2}{2} & D-\lambda \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$

$C-\lambda = \frac{w_S}{2} - \frac{w_I}{2} - \frac{J}{4} - \left(\frac{w_S}{2} - \frac{w_e}{2}\right) = -\frac{w_I}{2} - \frac{J}{4} + \frac{w_e}{2}$



$D-\lambda) = \frac{w_S}{2} + \frac{w_I}{2} + \frac{J}{4} - \left(\frac{w_S}{2} - \frac{w_e}{2}\right) = \frac{w_I}{2} + \frac{J}{4} + \frac{w_e}{2}$

$$\begin{pmatrix} -\frac{w_I}{2} - \frac{J}{4} + \frac{w_e}{2} & \frac{w_2}{2} \\ \frac{w_2}{2} & \frac{w_I}{2} + \frac{J}{4} + \frac{w_e}{2} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -w_I - \frac{J}{2} + w_e & w_2 \\ w_2 & w_I + \frac{J}{2} + w_e \end{pmatrix} = 0$$

$$\begin{pmatrix} \frac{-w_I - \frac{J}{2}}{w_e} + \frac{w_e}{w_e} & \frac{w_2}{w_e} \\ \frac{w_2}{w_e} & \frac{w_I + \frac{J}{2}}{w_e} + \frac{w_e}{w_e} \end{pmatrix} = \begin{pmatrix} c\theta+1 & s\theta \\ s\theta & -c\theta+1 \end{pmatrix} \begin{matrix} c_1 \\ c_2 \end{matrix} = 0$$

$c_1 s = -c_2(-c+1) = c_2(1-c$

$c_1 = -c_2 \frac{(1-c)}{s} = -c_2 T\left(\frac{\theta}{2}\right)$

$c_1(c+1) + c_2 s = 0$

$c_1 s + c_2(-c+1) = 0$

$c_1^2 + c_2^2 = 1 \quad c_2^2 + c_2^2 T^2(\theta/2) = 1 \quad c_2^2 = \frac{1}{1 + \frac{s^2}{c^2}}$

$c_2^2 = \frac{c^2}{c^2+s^2} = c^2 \therefore c_2 = c\left(\frac{\theta}{2}\right) \quad c_1 = c_2\left(\frac{s\frac{\theta}{2}}{c\,\theta/2}\right) = s\left(\frac{\theta}{2}\right)$

$c_1 = -s\left(\frac{\theta}{2}\right)$

$c_2 = c\left(\frac{\theta}{2}\right)$

$\lambda^-_B = \frac{w_S}{2} - \frac{w_e}{2}$

⑯

$$v_{24} = \frac{w_s}{2} + \frac{w_e}{2} - \left(-\frac{w_s}{2} + \frac{w_e}{0}\right) = \frac{w_s}{2} + \frac{w_e}{2} + \frac{w_s}{2} - \frac{w_e}{2} = w_s$$

$$A_{24} = (c_A s_A 0 0)\begin{pmatrix} 0110 \\ 0001 \\ 0001 \\ 0000 \end{pmatrix}\cancel{\begin{pmatrix} c \\ s \\ 0 \\ 0 \end{pmatrix}}\begin{pmatrix} 0 \\ 0 \\ c \\ s \end{pmatrix} = (c_A s_A 0 0)\begin{pmatrix} c \\ s \\ s \\ 0 \end{pmatrix} = c^2 + s^2 = 1$$

$$v_{34} = \frac{w_s}{2} + \frac{w_e}{2} - \left(\frac{w_s}{2} - \frac{w_e}{2}\right) = \frac{w_s}{2} + \frac{w_e}{2} - \frac{w_s}{2} + \frac{w_e}{2} = w_e$$

$$A_{34} = (0 0 -s c)\begin{pmatrix} 0110 \\ 0001 \\ 0001 \\ 0000 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ c \\ s \end{pmatrix} = (0 0 -s c)\begin{pmatrix} c \\ s \\ s \\ 0 \end{pmatrix} = -s^2$$

$$\left.\begin{array}{l} v_{12} = w_{eA};\ A_{12} = -s^2 \\ v_{13} = w_s;\ A_{13} = 1 \end{array}\right\} w_{eA} = \sqrt{\frac{J^2}{4} - w_I J + w_I^2 + w_2^2}$$

$$\left.\begin{array}{l} v_{24} = w_s;\ A_{24} = 1 \\ v_{34} = w_{eB};\ A_{34} = -s^2 \end{array}\right\} w_{eB} = \sqrt{\frac{J^2}{4} + w_I J + w_I^2 + w_2^2}$$

$v_{24} =$

$v_{14} = w_s + w_{ea} + w_{eb}$

$$v_{14} = \frac{w_s}{2} + \frac{w_{eB}}{2} - \left(-\frac{w_s}{2} - \frac{w_{eA}}{0}\right) = \frac{w_s}{2} + \frac{w_{eB}}{2} + \frac{w_s}{2} + \frac{w_{eA}}{2} = w_s + \frac{w_{eA}}{2} + \frac{w_{eB}}{2}$$

$$A_{14} = (-s_A c_A 0 0)\begin{pmatrix} 0110 \\ 0001 \\ 0001 \\ 0000 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ c_B \\ s_B \end{pmatrix} = (-s_A c_A 0 0)\begin{pmatrix} c_B \\ s_B \\ s_B \\ 0 \end{pmatrix} = \cancel{2s}^2$$

$c(\alpha\beta)$ $s(\beta\beta)$

$-s(\alpha\beta)$ $c(\beta\beta)$

$c(\alpha\alpha)$ $s(\beta\alpha)$

$-s(\alpha\alpha)$ $c(\beta\alpha)$

Me

~~c~~ $-s(\alpha\beta)$ $c(\beta\beta)$ $|4\rangle$

$c(\alpha\beta)$ $s(\beta\beta)$ $|3\rangle$

$-s(\alpha\alpha)$ $c(\beta\alpha)$ $|2\rangle$

$c(\alpha\alpha)$ $s(\beta\alpha)$ $|1\rangle$

them

Review!

SHAKA

$$A_{12} = \begin{pmatrix} CA & SA & 0 & 0 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} -SA \\ CA \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} CA & SA & 0 & 0 \end{pmatrix} \begin{pmatrix} CA \\ 0 \\ 0 \\ 0 \end{pmatrix} = CA^2$$

$$A_{13} - (CA\,SA \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ CB \\ SB \end{pmatrix} = \begin{pmatrix} CA & SA & 0 & 0 \end{pmatrix} \begin{pmatrix} CB \\ SB \\ SB \\ 0 \end{pmatrix} \quad CACB + SASB$$

$$A14 - \begin{pmatrix} CASA00 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -SB \\ CB \end{pmatrix} = \begin{pmatrix} CASA00 \end{pmatrix} \begin{pmatrix} -SB \\ CB \\ CB \\ 0 \end{pmatrix} = -CASB + SACB$$

$$A_{23} = \begin{pmatrix} -SA & CA & 0 & 0 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ CB \\ SB \end{pmatrix} = \begin{pmatrix} -SA & CA & 0 & 0 \end{pmatrix} \begin{pmatrix} CB \\ SB \\ SB \\ 0 \end{pmatrix} = -SACB + CASB$$

$$A24 = \begin{pmatrix} -SACA00 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -SB \\ CB \end{pmatrix} = \begin{pmatrix} -SA & CA & 0 & 0 \end{pmatrix} \begin{pmatrix} -SB \\ CB \\ 0 \\ 0 \end{pmatrix} = \begin{matrix} +SASB \\ +CACB \end{matrix}$$

⑮

$$\lambda_B^+ = \frac{w_s}{2} + \frac{w_{eB}}{2} \quad — \quad \overset{s(\frac{\theta}{2})}{\underset{c(\frac{\theta}{2})}{\cancel{c(\tfrac{\theta}{2})}}}|\alpha\beta\rangle + \overset{\cancel{c(\frac{\theta}{2})}}{\underset{s(\frac{\theta}{2})}{\cancel{s(\tfrac{\theta}{2})}}}|\beta\beta\rangle = |4\rangle$$

$$\lambda_B^- = \frac{w_s}{2} - \frac{w_{eB}}{2} \quad — \quad -s(\tfrac{\theta}{2})|\alpha\beta\rangle + c(\tfrac{\theta}{2})|\beta\beta\rangle = |3\rangle$$

$$\uparrow E$$

$$\lambda_A^+ = \frac{-w_s}{2} + \frac{w_{eA}}{2} \quad — \quad c(\tfrac{\theta}{2})|\alpha\alpha\rangle + s(\tfrac{\theta}{2})|\beta\alpha\rangle = |2\rangle$$

$$\lambda_A^- = \frac{-w_s}{2} - \frac{w_{eA}}{2} \quad — \quad -s(\tfrac{\theta}{2})|\alpha\alpha\rangle + c(\tfrac{\theta}{2})|\beta\alpha\rangle = |1\rangle$$

$$v_{12} = \frac{E_2 - E_1}{\hbar} = \frac{-w_s}{2} + \frac{w_e}{2}\left(-\frac{w_s}{2} - \frac{w_e}{2}\right) = \frac{-w_s}{2} + \frac{w_e}{2} + \frac{w_s}{2} + \frac{w_e}{2} = w_e$$

$$A_{12\,rc} = \left(\overset{-s}{\cancel{c}}\ \overset{c}{\cancel{s}}\ 0\ 0\right)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}+\cancel{c}\\ \cancel{s}\\0\\0\end{pmatrix} - \cancel{s}\,(-s\,c\,0\,0)\begin{pmatrix}s\\0\\0\\0\end{pmatrix} = -s^2$$

$$v_{13} = E_3 - E_1 = \frac{w_s}{2} - \frac{w_e}{2} - \left(-\frac{w_s}{2} - \frac{w_e}{2}\right) = \frac{w_s}{2} - \frac{w_e}{2} + \frac{w_s}{2} + \frac{w_e}{2} = w_s$$

$$A_{13} = \cancel{(-s\,c\,0\,0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}-s\\c\\0\\0\end{pmatrix} = (-s\,c\,0\,0)\begin{pmatrix}c\\0\\0\\0\end{pmatrix} = -cs}$$

$$A_{13} = (-s\,c\,0\,0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\-s\\c\end{pmatrix} = \overset{(-s\,c\,0\,0)}{\begin{pmatrix}-s\\c\\c\\0\end{pmatrix}} = s^2 + c^2 = 1$$

$$A_{13} = (-S_A\ C_A\ 0\ 0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\-S_B\\+C_B\end{pmatrix} = (\overset{-S_A}{\cancel{S_A}}\ C_a\ 0\ 0)\begin{pmatrix}-S_B\\+C_B\\+C_B\\0\end{pmatrix} = +S_A S_B + C_a C_B$$

OK use

(Me)

$$A_{12} = (-S_A\ C_A\ 0\ 0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}C_A\\S_A\\0\\0\end{pmatrix} = \begin{matrix}S_A\\0\\0\\0\end{matrix} = -S_A^2$$

$$A_{14} = (-S_A\ C_A\ 0\ 0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&\textcircled{0}\end{pmatrix}\begin{pmatrix}0\\0\\C_B\\S_B\end{pmatrix} = (-S_A\ C_A\ 0\ 0)\begin{pmatrix}C_B\\S_B\\S_B\\0\end{pmatrix} = -S_A C_B + C_a S_B$$

eq & op

$$A_{23} = (C_A\ S_A\ 0\ 0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\-S_B\\C_B\end{pmatrix} = (C_A\ S_A\ 0\ 0)\begin{pmatrix}-S_B\\C_B.\\C_B\\0\end{pmatrix} = -C_A S_B + S_A C_B$$

$$A_{24} = (C_A\ S_A\ 0\ 0)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\C_B\\S_B\end{pmatrix} = (C_A\ S_A\ 0\ 0)\begin{pmatrix}C_B\\S_B\\S_B\\0\end{pmatrix} = C_A C_B + S_A S_B$$

$$A_{34} = (0\ 0\ -S_B\ C_B)\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\begin{pmatrix}0\\0\\C_B\\S_B\end{pmatrix} = (0\ 0\ -S_B\ C_B)\begin{pmatrix}C_B\\S_B\\S_B\\0\end{pmatrix} = -S_B^2$$

~~$+C_B S_B$~~

~~$A_{24} = (0\,0\,C_B\,S_B)\begin{pmatrix}0110\\0001\\0001\\0000\end{pmatrix}\begin{pmatrix}C_A\\S_A\\0\\0\end{pmatrix} = (0\,0\,C_B\,S_A)\begin{pmatrix}S_A\\ \\ \\ \end{pmatrix}$~~

$$A_{24} = (C_A\,S_A\,0\,0)\begin{pmatrix}0110\\0001\\0001\\0000\end{pmatrix}\begin{pmatrix}0\\0\\C_B\\S_B\end{pmatrix} = (C_A\,S_A\,0\,0)\begin{pmatrix}C_B\\S_B\\S_B\\0\end{pmatrix} = C_A C_B + S_A S_B$$

$$A_{34} = (0\,0\,-S_B\,C_B)\begin{pmatrix}0110\\0001\\0001\\0000\end{pmatrix}\begin{pmatrix}0\\0\\C_B\\S_B\end{pmatrix} = (0\,0\,-S_B\,C_B)\begin{pmatrix}C_B\\S_B\\S_B\\0\end{pmatrix} = -S_B^2$$

$$v_{23} = \frac{w_s}{2} - \frac{w_{eB}}{2} - \left[-\frac{w_s}{2} + \frac{w_{eq}}{2}\right] = \frac{w_s}{2} - \frac{w_{eB}}{2} + \frac{w_s}{2} - \frac{w_{eq}}{2}$$

$$= w_s - w_{eB} - w_{eq}$$

$$v_{12} = w_{eq}$$

$$v_{13} = w_s - \frac{w_{eB}}{2} + \frac{w_{eq}}{2}$$

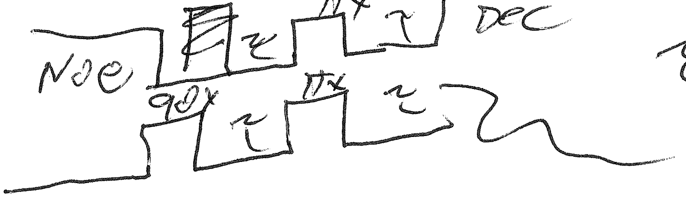
$$v_{14} = w_s + \frac{w_{eB}}{2} + \frac{w_{eq}}{2}$$

$$v_{23} = w_s - \frac{w_{eB}}{2} - \frac{w_{eq}}{2} \quad (\text{Me})$$

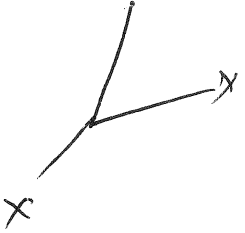
$$v_{24} = w_s + \frac{w_{eB}}{2} - \frac{w_{eq}}{2}$$

$$v_{34} = w_{eB}$$

APT –



$\tau = 1/2J$



IV-7

CH:

$S_x \xrightarrow[1/2J]{\pi J} c(\frac{Jt}{2})S_x + s(\frac{Jt}{2})I_zS_y$

$t = \frac{1}{J}$

$S_x c(\frac{J}{2}\cdot\frac{1}{J}\cdot 2\pi) + s(\pi)I_zS_y$

$-\overset{|}{\underset{|}{C}}-$ Just spin echo

$= -S_x$

$CH_2 =$ J_1 evolution on S
J_2 evolution on S

$S_x \xrightarrow{J_1} c(\frac{J_1t}{2})S_x + s(\frac{J_1t}{2})2I_{1y}S_z$

$\left(+ s(\frac{J_2t}{2})2I_{2y}S_z \right)$

$\xrightarrow{J_2} c(\frac{J_1t}{2})\left[S_x c(\frac{J_2t}{2})\right.$

$+ \overset{0}{\cancel{s(\frac{J_1t}{2})}}\left[\cancel{2I_{1y}S_z}\ 2(I_{1x}+I_{2x})S_z c(\frac{J_2t}{2}) + \cancel{I_{2y}}\cdot s(\frac{J_2t}{2})\right)$

$= c(\frac{J_1t}{2})c(\frac{J_2t}{2})S_x + c(\frac{J_1t}{2})\overset{0}{\cancel{s(\frac{J_2t}{2})}}2I_{2x}S_z$

$= (-1)(-1)S_x = +S_x$

CH_3: start from

$$C\left(\frac{J_1 t}{2}\right) C\left(\frac{J_2 t}{2}\right) S_x \xrightarrow{H_{J_3}} c_1 c_2 \left[C\left(\frac{J_3 t}{2}\right) S_x + \overset{0}{\cancel{2 I_{3y} S\left(\frac{J_3 t}{2}\right)}}\right]$$

$$(-1)(-1)(-1) S_x = -S_x$$

$c_1 = C\left(\frac{J_1 t}{2}\right)$ $\quad s_1 = S\left(\frac{J_1 t}{2}\right)$

$c_2 = C\left(\frac{J_2 t}{2}\right)$ $\quad s_2 = S\left(\frac{J_2 t}{2}\right)$ etc

$S\left(\frac{J t}{2}\right) = 0$

$$S_x \xrightarrow{J_1} c_1 S_x + \overset{0}{\cancel{s_1 2 I_{1z} S_x}}$$

$$c_1 S_x \xrightarrow{J_2} c_1 \left[c_2 S_x + \overset{0}{\cancel{s_2 2 I_{2z} S_x}}\right]$$

$$c_1 c_2 S_x \xrightarrow{J_3} c_1 c_2 \left[S_x c_3 + s_3 2 I_{3z} S_x\right]$$

$$= c_1 c_2 c_3 S_x = (-1)(-1)(-1) S_x = -S_x$$

The ~~[illegible]~~ sine part:

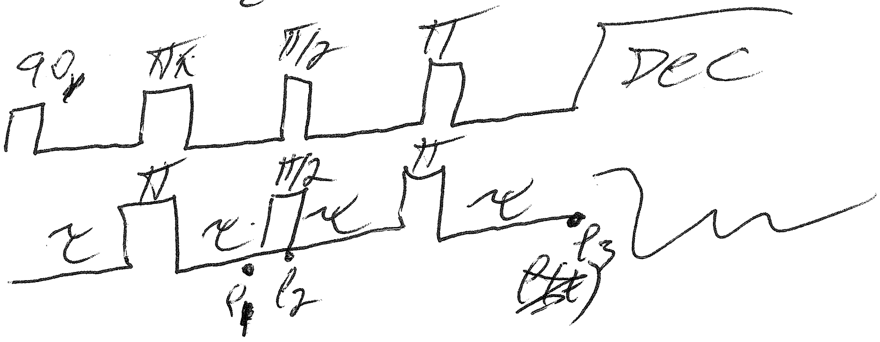
$$s_1 2 I_{1z} S_x \xrightarrow{J_2} s_1 \left[2 (I_{1z} + I_{2z}) S_x C\left(\frac{J_2 t}{2}\right) \cancel{+} - S_y S\left(\frac{J_2 t}{2}\right)\right]$$

$$= s_1 \left[2 (I_{1z} + I_{2z}) S_x c_2 \cancel{+} - S_y s_2\right.$$

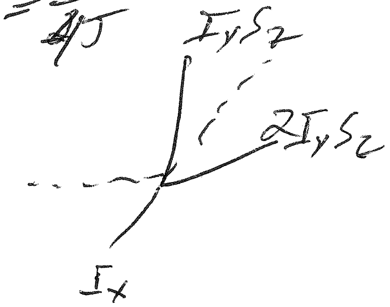
$$s_1 c_2 \left[2 (I_{1z} + I_{2z} + I_{3z}) c_3 - s_2 \left[S_y c_3 - 2 I_{3z} S_x s_3\right]\right.$$

$$0 = \overset{0}{\cancel{s_1 c_2}} c_3 \left[2 (I_{1z} + I_{2z} + I_{3z})\right) - s_2 c_3 S_y + 2 s_2 s_3 I_{3z} S_x$$

V-8) calc $\rho_{^{13}C}$ For refocussed inept



$\tau = \frac{1}{4J}$



$$\rho_x \xrightarrow{HJ} c\left(\frac{J}{2}\cdot t\right) I_x + s\left(\frac{Jt}{2}\right) 2I_yS_z \qquad t = 2\tau = \frac{1}{2J}$$

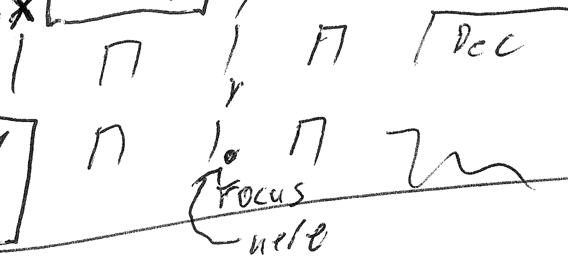
$$c\left(\frac{\pi}{2}\right) I_x + s\left(\frac{\pi}{2}\right) 2I_yS_z$$

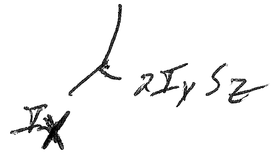
$$\rho_1 = 2I_yS_z \xrightarrow[\cancel{(\frac{\pi}{2})_{x,S}}]{(\pi/2)_{x},\, I} 2I_zS_z \xrightarrow{(\pi/2)S} \boxed{-2I_zS_x = \rho_2} \text{ AP}$$

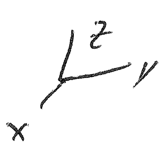
$$-2I_zS_x \longrightarrow -2I_zS_y \overset{0}{c\left(\frac{Jt}{2}\right)} + \overset{1}{s\left(\frac{Jt}{2}\right)} S_x \qquad \frac{Jt}{2} = \frac{\pi}{2}$$

$$\boxed{\rho_3 = S_x} \text{ IP}$$

① $-x$ $\boxed{V-9}$







Problem V-9

~~E~~ Drop S·term after 2 90's

CH_2^0: $I_x \xrightarrow{JI_zS_z} I_x C + 2I_yS_zS$

~~$S\,2I_yS_z \xrightarrow[90_xS]{90_xI} S\,2I_zS_y \xrightarrow{JI_zS_z} S[2I_zS_y C + -S_x S]$~~

~~$= CS\,2I_zS_y - S_x S^2$~~

keep For CH

$I_y \xrightarrow{JI_zS_z} CI_y + -S2I_xS_zS \xrightarrow[90_y]{90_x} +2I_zS_y S + \cancel{CI_z}$ Doesn't evolve

CH:

$S\,2I_zS_y \rightarrow S[2I_zS_x C + I_y S] = CS\,2I_zS_x + \boxed{SS\,I_y}$

OK

Spk. on 49

$Tr(I_xI_y) \neq 0 \quad \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} = \begin{pmatrix}0&0\\0&1\end{pmatrix} = Tr = 1$

$Tr(I_zI_xI_y) = 0 \quad \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&0\\0&1\end{pmatrix} = \begin{pmatrix}0&0\\0&0\end{pmatrix} = 0$

AP not Detected

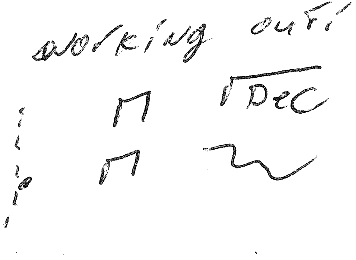
$I_y \xrightarrow{JI_zS_z} I_y C + -2I_xS_zS \xrightarrow[1]{1} 2I_zS_xS + I_zC$

$S2I_zS_x \xrightarrow{J} S[2I_zS_x C + S_y S] = CS\,\cancel{2I_zS_x}^{\,0}$

$+ \boxed{SS\;S_yI^+}$ — $Tr(S_yS_+ \neq 0)$

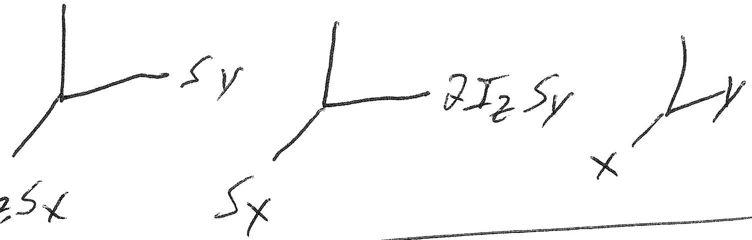
1-SPIN ⇒ $\boxed{S \cdot S_y}$

In essence working out:



— S~~tarting w/ spin~~ terms

CH_2 ② $\xrightarrow{J_1 I_{1z} S_z} \xrightarrow{J_2 I_{2z} S_z}$



Keep

$2I_{1z}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{1z}S_x C + S S_y$

$2I_{2z}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{2z}[S_x C + 2I_{1z}S_y S]$

$= 2I_{2z}S_x C + 4I_{2z}I_{1z}S_y S$

$S S_y \xrightarrow{J_2 I_{2z} S_z} S[S_y C - 2I_{2z}S_y S]$

$= \boxed{CSS_y} - S^2 2I_{2z}S_y$

$2I_{1z}S_x C \xrightarrow{J_2 I_{2z} S_z} 2I_{1z}C[S_x C + 2I_{2z}S_y S]$

$= C^2 2I_{1z}S_x + 4CS I_{1z}I_{2z}S_y$

$2C I_{2z}S_x \xrightarrow{J_2 I_{2z} S_z} C[2I_{2z}S_x C + S_y S]$

$= C^2 2I_{2z}S_x + \boxed{CSS_y}$

CH_2 - INEPT

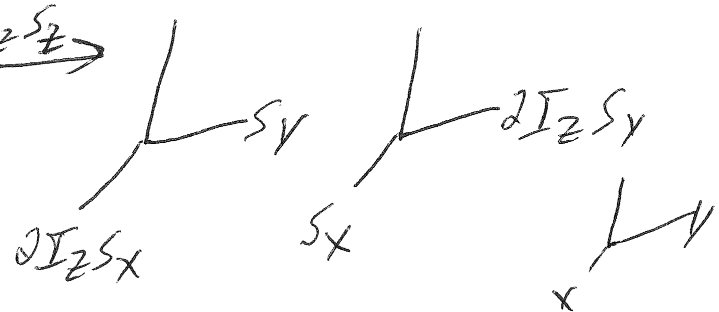
$$\boxed{\begin{aligned}\rho(t) &= CSS_y + CSS_y \\ &= 2CS\cdot S_y\end{aligned}}$$

$4S I_{1z}I_{2z}S_y = 2SI_{1z}(2I_{2z}S_y) \xrightarrow{J_2 I_{2z} S_z}$

$2SI_{1z}[2I_{2z}S_y C - S_y S]$

$= 4CS I_{1z}I_{2z}S_y - 2S^2 I_{1z}S_y$

CH_3 ③ $\xrightarrow{J_1 I_{1z} S_z} \xrightarrow{J_2 I_{2z} S_z} \xrightarrow{J_3 I_{3z} S_z}$



$$2(I_{1z}+I_{2z}+I_{3z})S_x = \rho_S$$

$$2I_{1z}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{1z}S_x c + s S_y$$

$$2I_{2z}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{2z}S_x c + 4 I_{2z} I_{1z} S_y s$$

$$2I_{3z}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{3z}S_x c + 4 I_{3z} I_{1z} S_y s$$

$$\xrightarrow{J_2 I_{2z} S_z} 2cs \cdot S_y$$

$$c[2I_{3z}S_x] \xrightarrow{J_2 I_{2z} S_z} 2c\,2I_{3z}[S_x c + 2I_{2z} S_y s]$$ — not observable

$$= c^2\, 2I_{3z}S_x + 4c I_{3z} I_{2z} S_y s$$

$$\xrightarrow{J_3 I_{3z} S_z}$$

$$c^2[2I_{3z}S_x] \xrightarrow{J_3 I_{3z} S_z} c^2[2I_{3z}S_x s + S_y s]$$

$$= c^2\, 2I_{3z}S_x s + \boxed{c^2 s \cdot S_y}$$ ①

Ⓐ Terms From $J_2 I_{2z} S_z$ eval.:

$$cs S_y \xrightarrow{J_3 I_{3z} S_z} cs[S_y c - 2 I_{3z} S_x s)$$

$$= \boxed{c^2 s S_y}^{③} - 2cs^2 I_{3z} S_x$$

$$-s^2 2 I_{2z} S_y \xrightarrow{J_3 I_{3z} S_z} -s^2 2 I_{2z} [S_y c - 2 I_{3z} S_x s)$$

$$c^2 2 I_{1z} S_x \xrightarrow{J_3 I_{3z} S_z} c^2 2 I_{1z} [S_x c + 2 I_{3z} S_y s)$$

$$4cs I_{1z} I_{2z} S_y \xrightarrow{J_3 I_{3z} S_z} 4cs I_{1z} I_{2z} [S_y c - 2 I_{3z} S_x s]$$

$$c^2 2 I_{2z} S_x \xrightarrow{J_3 I_{3z} S_z} c^2 2 I_{2z} [S_x c + 2 I_{3z} S_y s]$$

$$cs \cdot S_y \xrightarrow{J_3 I_{3z} S_z} cs[S_y c - 2 I_{3z} S_x s)$$

$$= \boxed{c^2 s \cdot S_y}^{②} - 2cs^2 I_{3z} S_x$$

$$4cs I_{1z} I_{2z} S_y \xrightarrow{J_3 I_{3z} S_z} 4cs I_{1z} I_{2z} [S_y c - 2 I_{3z} S_x s]$$

$$-2s^2 I_{1z} S_y \xrightarrow{J_3 I_{3z} S_z} -2s^2 I_{1z} [S_y c - 2 I_{3z} S_x s]$$

$$\boxed{\rho(t) = 3 * c^2 s * S_y}$$ — 3 spin Inept

⑤ 3 spin inept summary

$\tau = \frac{1}{4J} \Rightarrow S(\frac{Jt}{2}) = 1$

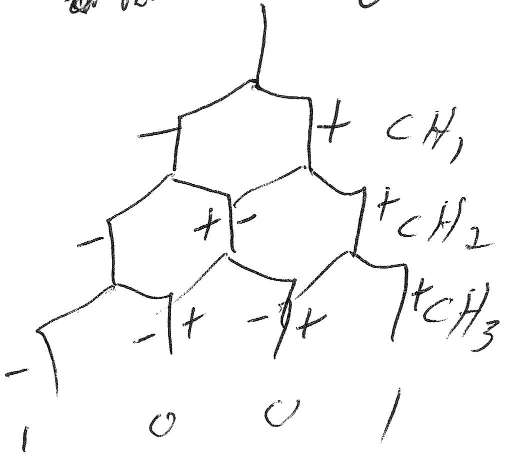
~~$C(\frac{Jt}{2})$~~

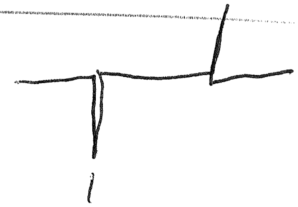
CH: $S \cdot S_y$

CH$_2$: $2C S \cdot S_y$

CH$_3$: $3C^2 S \cdot S_y$

For sine func





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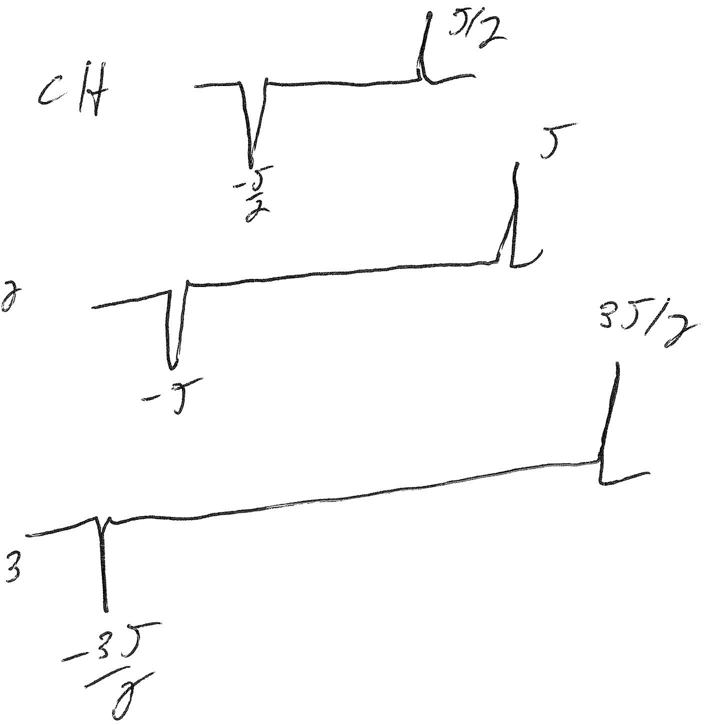
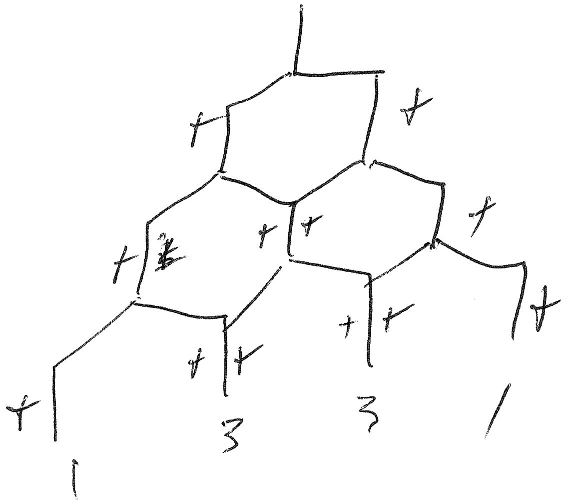
* Put into simulation should get



For cos

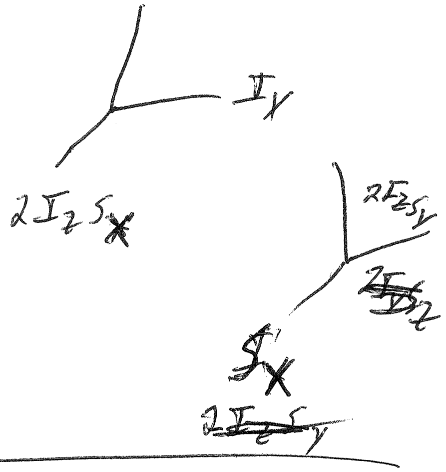


Keep

$$2I_{1z}S_x \xrightarrow[J_2 I_{2z} S_z]{} 2I_{1z}\left[S_x C\left(\frac{J_2 t}{2}\right) + 2I_{2z} S_y S\left(\frac{J_2 t}{2}\right)\right]$$

Keep

$$2I_{2z}S_x \xrightarrow[J_1 I_{1z} S_z]{} 2I_{2z}\left[S_x C\left(\frac{J_1 t}{2}\right) + 2I_{1z} S_y S\left(\frac{J_1 t}{2}\right)\right]$$



$$2I_{1x}S_z \xrightarrow{J_1 I_{1z} S_z} 2\cancel{\left[I_{1x} C\left(\frac{Jt}{2}\right) +\right.} 2I_{1x}S_z C\left(\frac{J_1 t}{2}\right) + I_{1y} S\left(\frac{J_1 t}{2}\right)$$ as befor

$$2I_{1x}S_z \xrightarrow{J_2 I_{2z} S_z} 2I_{1x}S_z$$ [Doesn't evolve.]

MQ Problem Too Before →

$$2I_yS_x \xrightarrow{J I_z S_z} 2I_yS_x$$ [Doesn't evolve]

---

however NOW →

$$2I_{1y}S_x \xrightarrow{J_2 I_{2z} S_z} 2I_{1y}\left[S_x C\left(\frac{J_2 t}{2}\right) + 2I_{2y} S_z S\left(\frac{J_2 t}{2}\right)\right]$$

$$= 2I_{1y}S_x C\left(\frac{J_2 t}{2}\right) + 4I_{1y}I_{2y}S_z S\left(\frac{J_2 t}{2}\right)$$

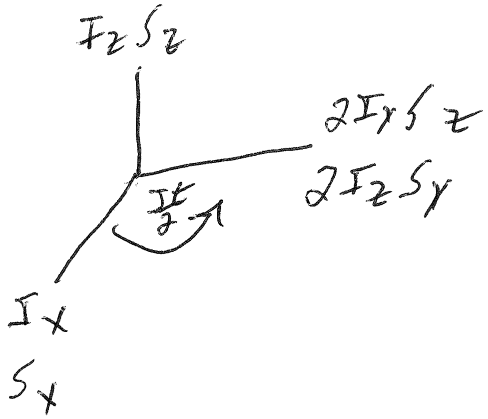
NOW →

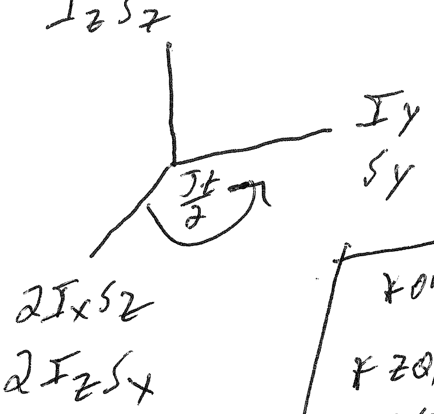
$$2I_{2y}S_x \xrightarrow{J_1 I_{1z} S_z} 2I_{2y}\left[S_x C\left(\frac{J_1 t}{2}\right) + 2I_{1y} S_z S\left(\frac{J_1 t}{2}\right)\right]$$

$$= 2I_{2y}S_x C\left(\frac{J_1 t}{2}\right) + 4I_{2y}I_{1y}S_z S\left(\frac{J_1 t}{2}\right)$$

$\rho(t) = e^{-iH_J t} \rho(0) e^{iH_J t}$ $H_J = J I_z S_z$

$[I_x, 2I_yS_z) = i I_zS_z$





J-coupl

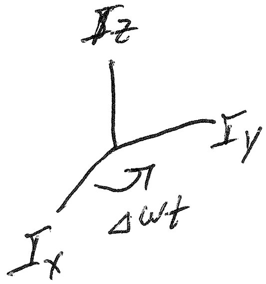
* only SQ terms evolve

* ZQ, DQ Do not evolve under H_J

Chem shifts

$[H_{cs}, H_J] = 0$ ∴ order Doesn't matter

$$e^{-iH_{cs}t} e^{-iH_J t} = e^{-iH_J t} e^{-iH_{cs}t}$$



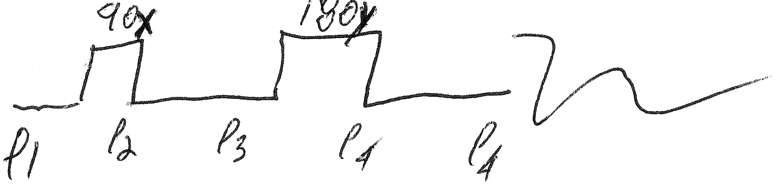
$I_x \xrightarrow{H_{cs}} I_x C(\Delta\omega t) + I_y S(\Delta\omega t)$

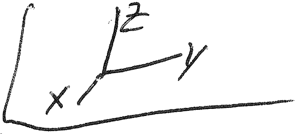
* I_xS_z evolve like independent spins

$R I_x R^- R I_z R^-$

∴ DQ, SQ all evolve H_{cs}

I_z Doesn't





$$\boxed{H_r = -\hbar(\omega_0-\omega)I_z - \hbar\omega_1 I_{x,y}}$$

$$\boxed{e^{-iH_r t/\hbar}\rho(0)e^{iH_r t/\hbar} = \rho(t)}$$

$$= e^{i\omega_1 t I_x} I_z e^{-i\omega_1 t I_x} = \rho(t)$$

$$= U_x(\omega_1 t) I_z U_{-x}(\omega_1 t) = \rho(t)$$

$\rho_1 = I_z$ $\quad \rho_2 = U_x I_z U_x^- = I_z c(\phi) - I_y s(\phi)$ $\quad \phi = \pi/2$

Pulse

$\rho_2 = -I_y$

$H_{cs} = -\Delta\omega I_z$ $\quad \Delta\omega = \omega_0 - \omega$

$\rho_2 \xrightarrow{H_{cs}} e^{-iH_{cs}t}\rho_2 e^{iH_{cs}t} = \rho_3$ $\quad \phi = (\Delta\omega t)$

$U_z(-I_y)U_z^- = -[I_y c(\phi) - I_x s(\phi)]$

$\rho_3 = -I_y c + I_x s$

$\phi = \pi$

$\rho_4 = U_R \rho_3 U_R^- \rightarrow -c[U_y I_y U_y^-] = -cI_y$

$+s[U_y I_x U_y^-]$ $\quad +s[I_x c(\pi) + I_z s(\pi)]$

~~$\rho_4 = -c(\pi)I_y$~~ $\quad \rho_4 = -c(\Delta\omega t)I_y + s(\Delta\omega t)[-I_x]$

$\rho_4 = -I_y c - I_x s$

$$\rho_5 \xrightarrow[\rho_4]{H_{cs}} \begin{matrix} -c[I_y c - I_x s] \\ -s[I_x c + I_y s] \end{matrix} = -c^2 I_y + cs\cancel{I_x} - cs\cancel{I_x} - s^2 I_y$$

$$\rho_5 = -I_y(c^2+s^2) = \boxed{\begin{matrix} -I_y = \rho_5 \\ = \rho_2 \end{matrix}}$$