II.17 Problems

1) $\boxed{\text{Voltage} = \mathcal{E} = \frac{d\phi}{dt}}$: magnetic flux $\approx w_0 \cdot M_0 \cdot A$

w_0 = larmor

M_0 = equil. magnetization

A = Area of coil

* solve for a voltage which should be $\approx 10^{-4} \to 10^{-5}$ volts

$w_0 = \gamma B_0$

$$M_0 = \frac{N\gamma^2\hbar^2 B_0}{4KT} = \frac{N\gamma\hbar^2 w_0 \pi}{2KT}$$

$$\mathcal{E} = \frac{N\gamma\hbar^2 w_0 A\pi}{2KT} = \frac{10^{20}\times 26.75\times 10^{7}\times 1.05^{2}\times 10^{-68}}{2\times 1.38\times 10^{-23}\times 300}$$

$$\frac{300^2\times 10^{12}\times \hbar^2\times 0.05^2}{} = 8.0\times 10^{-5}\,V$$

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2) The signal is : $\boxed{S_+ = \frac{\gamma^3\hbar^2 N B_0^2}{4KT} e^{i w_0 t}}$

* IN $CHCl_3$, The Ratio of ^1H & ^{12}C is 1:1

* The Abundance of ^{13}C is 1.1%

* Let B = field for ^{13}C

$$1.1\% \times \gamma_{^{13}C}^{3} \times B^2 = \gamma_{^1H}^{3} \times 2.35^2$$

$$B = \left(\frac{\gamma_{1H}}{\gamma_{13C}}\right)^{3/2} \times \frac{1}{(1.1\%)^{1/2}} \times 2.35 = 179 \text{ Tesla}$$

* If molecules are ^{13}C enriched then

$$B = \left(\frac{\gamma_{1H}}{\gamma_{13C}}\right) \times 2.35 = 18.8\,T$$

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3) A DBM output signal is:

$$\boxed{\theta \propto S_1(t)\cdot S_2(t) = \cos(w_1 t + \phi_1)\cdot \cos(w_2 t + \phi_2)}$$

$$\theta = \frac{1}{2}\cos[(w_1 - w_2)t + (\phi_1 - \phi_2)] + \frac{1}{2}[\cos(w_1 + w_2)t + (\phi_1 + \phi_2)]$$

a)
* Note output frequencies: $(w_1 - w_2)$ & $(w_1 + w_2)$
" " phases: $(\phi_1 - \phi_2)$ & $(\phi_1 + \phi_2)$
∴ is sensitive to input phases/signal

b) Let $w_1 = w_2$ & output signals are 0 & $2w_1$

* filter out DC & left w/ $2w_1$

c) If we let $S_2 = \cos(180°) = -1$, the output signals are $\cos(w_1 t + \phi_1 - 180°)$. ∴ Input signal is 180° phase shifted.

D) If we apply a low level signal at the R-port & the RF at L-port, the output has a frequency difference & lower signal level

e) When a bias current flows in the X-port, the DBM passes RF. When bias is off, no current flows.

7
0

4) If we change the magnetic field direction "sufficiently" slowly; the magnetization follows the direction of the magnetic field.

In the rotating frame, the effective field has the components:

$$B_0 - \frac{\omega}{\gamma} \} \text{ along } \hat{z}$$

& B_1 is on x-y plane,

* If the resonance frequency ω is swept slowly from well below & well above the exact resonance frequency

eg) $\frac{\omega}{\gamma} \ll B_0$ to $\frac{\omega}{\gamma} \gg B_0$,

we can see the component along $\hat{z}$ change gradually from $+\hat{z}$ to $-\hat{z}$, while the component on x-y is unchanged.

7

5) see p78

6) Inverse Fourier Transform is:

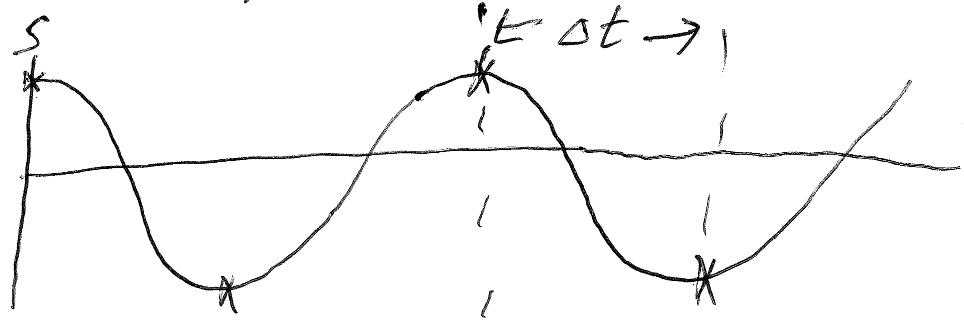
$$\boxed{S(t) = \int_0^{\infty} F(\omega)\, e^{i\omega t}\, d\omega}$$

The Discrete Fourier Transform:

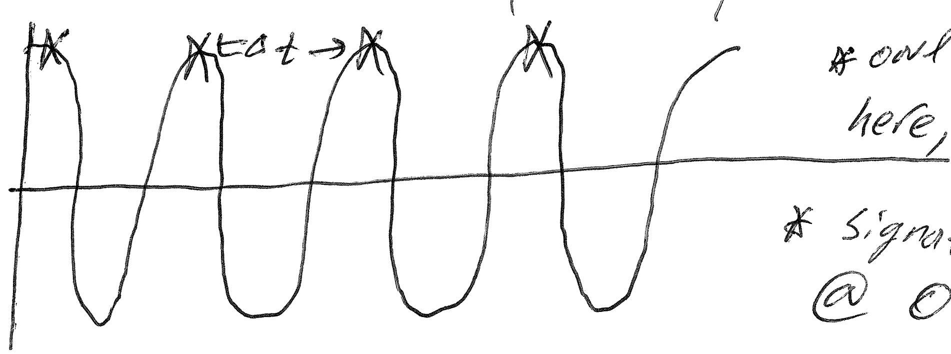
$$\boxed{S_m = \sum_{K=0}^{N-1} F_K\, e^{i 2\pi K m / N}}$$

7) i) 2 pts are minimum pts needed per cycle for assigning the resonance frequency.

ii)



$v = \frac{SW}{2}$



* only 1 pt/cycle here, under sampled

* signal appears @ 0

iii) $v = -SW$, resonance will appear at zero.

* Resonance frequency btw $\frac{SW}{2}$ & SW will appear btw $\frac{SW}{2}$ & zero.

* If Δt is wrong, signals appear at $\boxed{\frac{1}{\Delta t} - v}$

v = actual freq

8) The dwell time is:

$$\Delta t = \frac{1}{SW} = \frac{1}{10 KHz} = 0.1 ms$$

Digital Resolution of 0.5 Hz give AT:

$$AT = \frac{1}{0.5 Hz} = 2 S$$

∴ Points needed are:

$$\frac{AT}{\Delta t} = \frac{2 \times 10^3}{0.1} = 2 \times 10^4$$ { for FT algorithm use 32,768 Pts

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9) $$\boxed{S(t) = S_0 e^{i\omega t} e^{-t/T_2} = S_0 e^{-n\Delta t} e^{i m \Delta \omega \Delta t}}$$

$\nu = 1000 Hz$; $\Delta t = 0.5 ms$; $\frac{1}{\pi T_2} = 1 Hz$; $T_2 = 0.318 S$

$$S_n = S(0) e^{-i 2\pi \Delta\nu n \Delta t} e^{-n\Delta t / T_2} = e^{-i n \pi} e^{-n \times 1.57 \times 10^{-3}}$$

$S(0) = S_0 \cdot 1$

$$S(1) = S_0 e^{-i 2\pi \times 1000 \times 1 \times 0.5 \times 10^{-3}} e^{-1 \times 0.5 \times 10^{-3} / 0.318}$$

$= \cancel{S_0 \, 0.9969} = 0.998 \cdot S_0$

$$S(2) = S_0 e^{-2\pi i} e^{-3.14 \times 10^{-3}} = \cancel{0.994 S_0}$$ $0.9969 S_0$

$$S(3) = S_0 e^{-3\pi i} e^{-4.11 \times 10^{-3}}$$

$= 0.994 \cdot S_0$

↓ and so on

10) After a $\pi/2$ pulse, the magnetization is on the x-y plane. Afterwords the magnetization goes as:

$$M_{x,y} = M_0 e^{-t/\tau_2}$$

& magnetization along z-axis is:

$$M_z = (1 - e^{-t/\tau_1}) M_0$$

$\therefore$ Total length M:

$$M = \sqrt{M_{x,y}^2 + M_z^2} = M_0 \sqrt{e^{-2t/\tau_2} + (1 - e^{-t/\tau_1})^2}$$

$$= M_0 \sqrt{1 - 2e^{-t/\tau_1} + e^{-2t/\tau_1} + e^{-2t/\tau_2}}$$

$$\boxed{M = M_0 (1 - 2e^{-t/\tau_1} + 2e^{-2t/\tau_2})^{1/2}}$$; where $\tau_1 = \tau_2$

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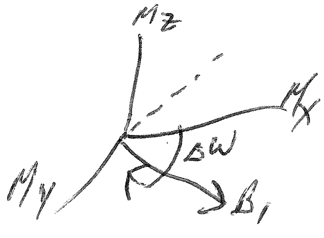
11) $$\frac{dM_x}{dt} = \gamma(M_y B_0 + M_z B_1 \sin \omega t) - \frac{M_x}{\tau_2}$$

?

$$\frac{dM_y}{dt} = \gamma(M_z B_1 \cos \omega t - M_x B_0) - \frac{M_y}{\tau_2}$$

$$\frac{dM_z}{dt} = \gamma(-M_x B_1 \sin \omega t - M_y B_1 \cos \omega t) - \frac{(M_z - M_0)}{T_1}$$



$$\boxed{\omega = -\gamma B_0}$$

12) $F(w) = \frac{1}{T}\int_{-\infty}^{\infty} s(t) e^{-iwt}\,dt$

$$= \frac{1}{T_2}\int_{-\infty}^{\infty} e^{i\Delta w t} e^{-t/T_2} e^{-iwt}\,dt$$

$\boxed{\text{recall: } s(t) = e^{i\Delta w t} e^{-t/T_2}}$

$$= \frac{1}{T_2}\int_{-\infty}^{\infty} e^{(i\Delta w - iw - 1/T_2)t}\,dt$$

$$= \frac{1}{T_2}\frac{1}{i(\Delta w - w) - 1/T_2} = \frac{1}{T_2}\cdot\frac{i(\Delta w - w) + 1/T_2}{-(\Delta w - w)^2 - 1/T_2^2}$$

$$\boxed{F(w) = \frac{T_2}{1+(w-\Delta w)^2 T_2^2} + \frac{iT_2^2(w-\Delta w)}{T_2^2(w-\Delta w)^2+1}}$$

$\Downarrow$ ABSORTIVE $\Downarrow$ DISPERSIVE

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i) $A(w) = \frac{1}{1+(w-\Delta w)T_2^2}$ when $w=\Delta w$ $A(w)$ has a maximum

$A(w)=1$

The Half-height will be $A(w)=1/2$

$$\frac{1}{1+(w-\Delta w)T_2^2} = 1/2$$

$$T_2^2(w-\Delta w)^2 = 1 \Rightarrow w_\pm = \Delta w \pm 1/2$$

$$\therefore \text{Fwhh} = w_+ - w_- = \frac{2}{T_2}$$

ii) The magnitude is : $\sqrt{A(\omega)^2 + D(\omega)^2} \stackrel{!}{=} \frac{1}{\sqrt{1+(\omega-\Delta\omega)^2 T_2^2}}$

when $\omega = \Delta\omega$, have max $= 1$

$\therefore$ at FWHH ; $\frac{1}{\sqrt{1+(\omega-\Delta\omega)^2 T_2^2}} = \frac{1}{2}$

$$(\omega - \Delta\omega)^2 T_2^2 = 3$$

$$\omega_\pm - \Delta\omega = \pm \frac{\sqrt{3}}{T_2}$$

$$\omega_\pm = \Delta\omega \pm \frac{\sqrt{3}}{T_2}$$

$\therefore$ The FWHH $\Rightarrow \omega_+ - \omega_- = \frac{2\sqrt{3}}{T_2}$

* comparison of magnitude vs. phased FWHH reveals the phased is narrower by a factor of $\sqrt{3}$!

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14) Phase collection eqn: $\boxed{\phi = \theta + \Delta\omega \cdot \tau}$

on resonance : $\Delta\omega = 0$

$\phi = 45^\circ$

From sequence: $\tau = 50\mu s + 100\mu s = 150\mu s$

i) $\Delta\omega = 5\,kHz$

$$\phi = 45^\circ + 5\,kHz \times 150\mu s \times \left(\frac{360^\circ}{cycle}\right) = 315^\circ$$

14) ii) $\Delta W = -5KHz$

$$\phi = 45^\circ - 5KHz \times 150\mu s \times \frac{360^\circ}{cycle} = -225^\circ$$

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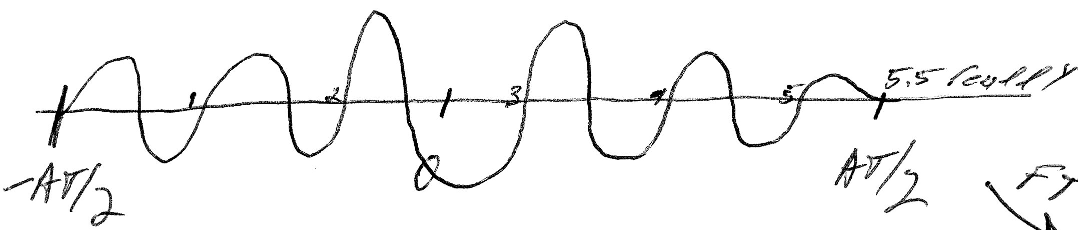
15) The $AT = 1024 \times 500\mu s = 512{,}000\mu s$

The Resonance Freq. $= \dfrac{6}{512{,}000\mu s} = 11\,Hz$

eg) 

$\approx 6\,\frac{cycles}{AT}$

* Assume both IR & II signal is collected

* 180° Phase collection needed to get $A(w)$

* by using linear Phase collection given by $\tau = AT/2$, like a dead time correction we get $A(w)$

$$\therefore \tau = \frac{180^\circ}{11.7 \times 360^\circ} = 42.7\,ms$$

? OR IS IT $\boxed{\phi = 180 \cancel{+} \frac{512{,}000\mu s}{2} * 11.7 * \frac{360^\circ}{cycle}}$

Think its this really

16) Since signal is linearly proportional to sample concentration & $\sqrt{\ }$ dependent on noise:

$$\frac{10}{1 \times \sqrt{1}} = \frac{100}{0.2 \times \sqrt{\text{scans}}} \Rightarrow \text{Scans} = 2500$$

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17) Lets use 0_- & 0_+ to represent time before & after the pulse β.

Z-magnetization after pulse is:

$$M_z(0_+) = M_z(0_-)\cos(\beta)$$

Z-magnetization after acquisition AT:

$$M_z(AT) = M_z(0_+)e^{-AT/T_2} + M_0(1 - e^{-AT/T_1})$$

Since $T_1 = T_2$

$$M_z(AT) = M_z(0_+)e^{-AT/T_1} + M_0(1 - e^{-AT/T_1})$$

when equilibriums reached: $M_z(AT) = M_z(0_-)$

$$M_z(0_-) = M_0 \frac{1 - e^{-AT/T_1}}{1 - e^{-AT/T_1}\cos(\beta)}$$

however, detect $M_{x,y}$ projection

$$M_{x,x}(0_+) = M_0 \left(\frac{1 - e^{-AT/T_1}}{1 - e^{-AT/T_1}}\right)\sin(\beta)$$

cont., we have the max at an angle when

$$\frac{dM_{x,y}(0_+)}{d\beta} = 0$$

$$\therefore \frac{dM_{x,y}(0_+)}{d\beta} = M_0(1-e^{-AT/T_1})\left[\frac{\cos(\beta)}{1-e^{-AT/T_1}\cdot\cos(\beta)} - \frac{e^{-AT/T_1}\sin^2(\beta)}{(1-e^{-AT/T_1}\cos(\beta))^2}\right] = \frac{M_0(1-e^{-AT/T_1})(\cos\beta - e^{-AT/T_1})}{1-e^{-AT/T_1}\cos(\beta)} = 0$$

$$\therefore \boxed{\cos(\beta) = e^{-AT/T_1}} \checkmark$$

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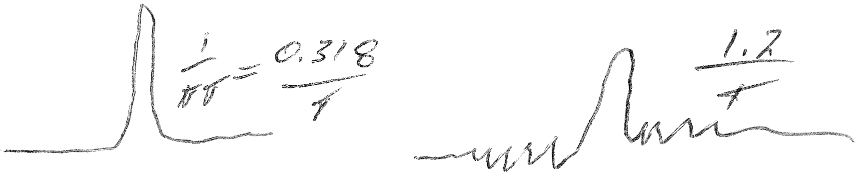
18) Let T = acquisition time for a fully collected FID.

$$\therefore \text{fwhh} = \frac{1}{\pi T_{\cancel{2}}} = \frac{0.318}{T_{\cancel{2}}}$$

* If we collect only $T/4$ we get $\frac{\sin(x)}{x} = \text{sinc}$

$$\therefore \text{fwhh} = \frac{1}{\pi T_2} = \frac{0.318}{T/4} \approx \frac{1.2}{T}$$



* can use LB to smooth out sinc, but loose resolution

19) create a vortex, effects shims/susceptibility

20) $\mathbb{R}$: $s(t) = \sin(\omega t) = \dfrac{-i\left(e^{i\omega t} - e^{-i\omega t}\right)}{2}$

$$\left.\begin{aligned} e^{i\omega t} &\Rightarrow A^+ + D^+ \\ e^{-i\omega t} &\Rightarrow A^- + D^- \end{aligned}\right\} \quad \frac{i\left(e^{-i\omega t} - e^{i\omega t}\right)}{2}$$

$$\Downarrow$$

$$\frac{i\left(A^- + D^- - A^+ - D^+\right)}{2}$$

$$\boxed{F(\omega) = i\left(\frac{A^- - A^+}{2}\right) + \frac{D^+ - D^-}{2}}$$

ii) $s(t) = i\cos(\omega t) = i\left(e^{i\omega t} + e^{-i\omega t}\right)/2$

$$= i\left(A^+ + D^+ + A^- + D^-\right)/2$$

$$\boxed{F(\omega) = i\left(\frac{A^+ + A^-}{2}\right) - \frac{D^+ + D^-}{2}}$$

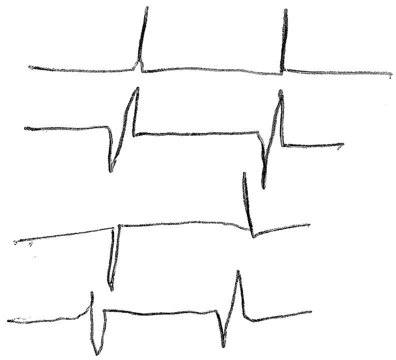
21) $F(\omega) = A(\omega) + i\, D(\omega) \Rightarrow$ detects I_-

Recall: R ∿ : cos $\xrightarrow{FT}$

II ——— : 0

R ——— : 0 $\xrightarrow{FT}$

II ∿ : sin



* FOR DETECTING I_+ or component oscillating @ $\Delta\omega$, simply acquire −sin in II.

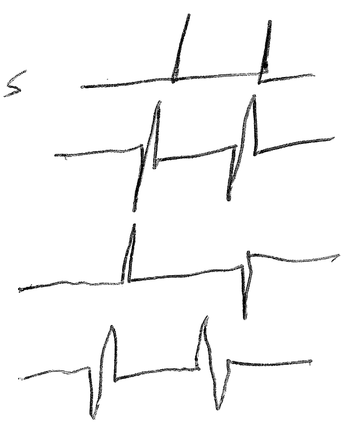
$\therefore$ $\boxed{F(\omega) = A(\omega) - i\, D(\omega)}$ [can show w/ Q.M. too]

Now R : cos

II : 0

R : 0

II : −sin



?

* Do w/ phase cycling & Receiver (ADC buffers)

expt	ADC 1 (R)	ADC 2 (II)	buffer (I)	buffer (II)
1	cos	sin	ADC 1	−ADC 2
2	−sin	cos	ADC 2	ADC 1
3	−cos	−sin	−ADC 1	ADC 2
4	sin	−cos	−ADC 2	−ADC 1

4 cos
−4 sin

22) i) DYNAMIC Range of a 12-BIT Digitizer:

$$\frac{2^{12}}{2} = 2^{11} = 2048$$

DYNAMIC Range of a 16-BIT Digitizer

$$\frac{2^{16}}{2} = 2^{15} = \cancel{5?}32768$$

ii) The maximum signal voltage should be set close to 5V, but not to exceed.

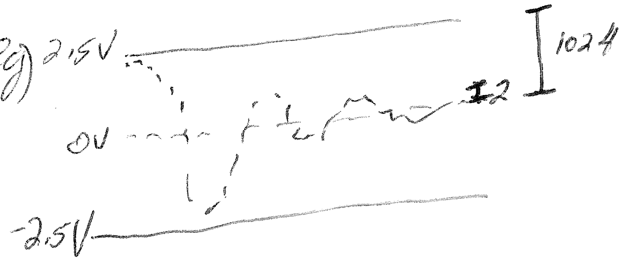
iii) a) $S/N = \infty$ ∴ # of scans needed is:

$$\frac{10^{23}}{10^{15}} = 256$$

b) $\frac{S}{N} \approx 0$ ∴ # of scans needed is:

$$\left(\frac{10^{23}}{10^{15}}\right)^2 = 65,536$$

eg

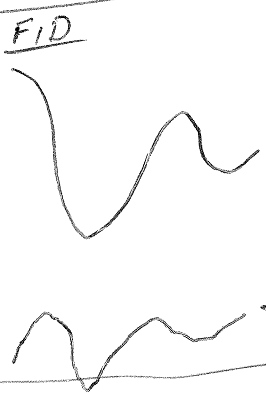


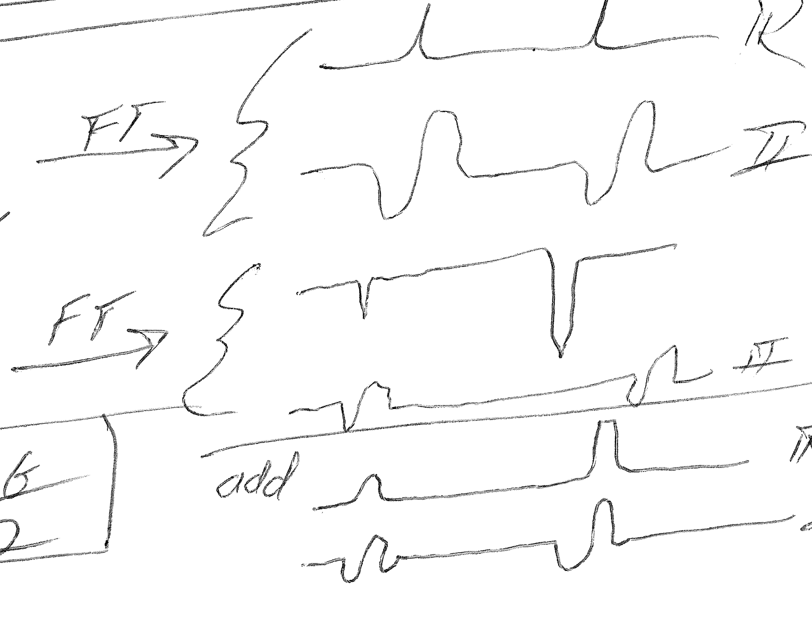
; defines when saturation sets in!

?

23) Larger signal R

smaller signal I





$\boxed{\text{Ghost intensity: } \frac{\Delta G}{2}}$

24)

ϕ_{TX}	IR signal	IF signal
0	$c(\Delta\omega t)$	$s(\Delta\omega t)$
90°	$-s(\Delta\omega t)$	$c(\Delta\omega t)$
180°	$-c(\Delta\omega t)$	$-s(\Delta\omega t)$
270°	$s(\Delta\omega t)$	$-c(\Delta\omega t)$

* see pg 63 regarding phase factor ϕ

25)
$$\boxed{\phi_{TX} = \phi_{TX}^{0} + \frac{2\pi(i-1)}{N}\,; \quad i = 1, 2, \ldots, N \qquad \phi_{TX}^{0} = 0 \to 360^\circ}$$

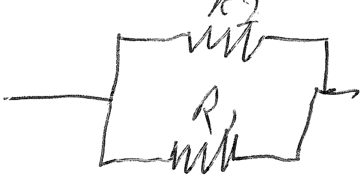
$$c_1^i = \cos[2\pi(i-1)/N] \qquad c_2^i = -\sin[2\pi(i-1)/N]$$

$$c_3^i = \sin[2\pi(i-1)/N] \qquad c_4^i = \cos[2\pi(i-1)/N]$$

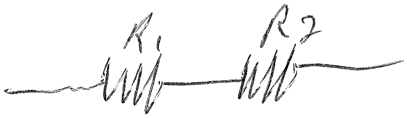
Let $N = 4$ (cyclops)

wrong

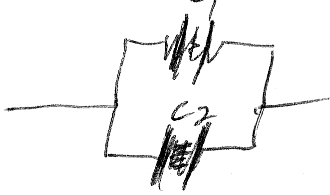
	$i=1$	$i=2$	$i=3$	$i=4$	?
$c_1^1 =$	1	0	−1	1	1
$c_2^1 =$	0	−1	0	0	
$c_3^1 =$	0	1	0	0	
$c_4^1 =$	1	0	−1	1	

26) i) 

$R = \frac{R_1 R_2}{R_1 + R_2}$ $\quad Z = \frac{R_1 R_2}{R_1 + R_2}$

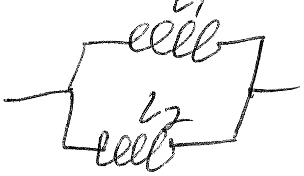


$R = R_1 + R_2$ $\quad Z = R_1 + R_2$

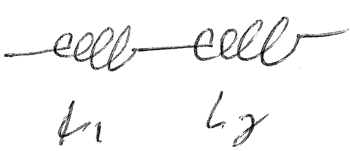
ii) 

$C = C_1 + C_2$ $\quad Z = \frac{-i}{wC} = \frac{-i}{w(C_1 + C_2)}$

$C = \frac{C_1 C_2}{C_1 + C_2}$ $\quad Z = \frac{-i}{wC} = \frac{-i(C_1 + C_2)}{w(C_1 C_2)}$

iii) 

$L = \frac{L_1 L_2}{L_1 + L_2}$ $\quad Z = iwL = iw\left(\frac{L_1 L_2}{L_1 + L_2}\right)$



$L = L_1 + L_2$ $\quad Z = iw(L_1 + L_2)$

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27) i) $Z_{eq} = R + iwL - \frac{i}{wC}$ $\quad Z_{eq} = \sqrt{R^2 + \left(iwL + \frac{i}{wC}\right)^2}$

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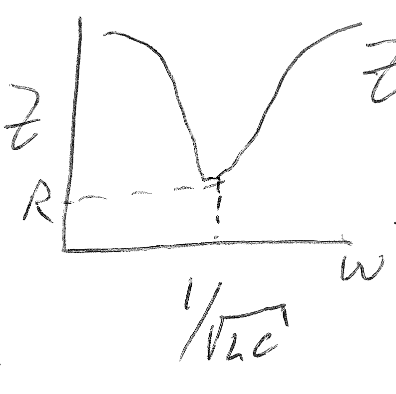
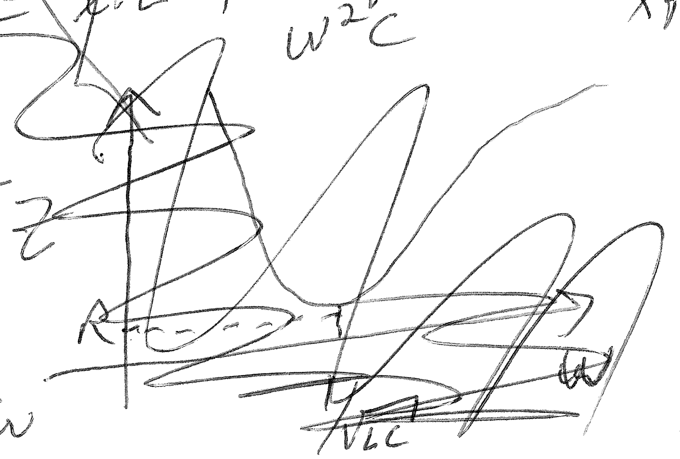
27 ii) $\cancel{\frac{\partial Z}{\partial w}} = iL + \frac{i}{w^2 C} = 0$ $\quad X_T = \frac{iwL}{w^2 CL + 1}$





$Z = \frac{R \cdot X_T}{\sqrt{R^2 + X_T^2}}$

?

(16)

29) The output power is:

$$\boxed{Z_s = -Z_\ell}$$ shouldn't it be ?

$$P_{out} = \frac{V^2 Z_\ell}{(Z_s + Z_\ell)^2} = \frac{V^2}{\left(\frac{Z_s}{\sqrt{Z_\ell}} + \sqrt{Z_\ell}\right)^2}$$

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30) i) Ideally $V = 0$

ii) $V = 0$

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31) Let Z_1 = impedance of tuning circuit

$$\therefore \frac{1}{Z_1} = i\omega C_2 + \frac{1}{(r + i\omega L)}$$

note: $(a+ib)(a-ib) = a^2 + b^2$

$$= \frac{r}{r^2 + \omega^2 L^2} + i\left[\omega C_2 - \frac{\omega L}{r^2 + \omega^2 L^2}\right]$$

$$\therefore Z_1 = \frac{r/(r^2+\omega^2 L^2) - i\left[\omega C_2 - \omega L/(r^2+\omega^2 L^2)\right]}{r^2/(r^2+\omega^2 L^2)^2 + \left[\omega C_2 - \omega L/(r^2+\omega^2 L^2)\right]^2}$$

The total impedance is:

$$\boxed{Z = Z_1 + \frac{1}{i\omega C_1}}$$

at the resonance condition, $\mathbb{I}$ part is zero

$$\mathbb{I}: \frac{-\left[\omega C_2 - \omega L/(r^2+\omega^2L^2)\right]}{r^2/(r^2+\omega^2L^2)^2 + \left[\omega C_2 - \omega L/(r^2+\omega^2L^2)\right]} - \frac{1}{\omega C_1} = 0$$

$$\mathbb{R}: \frac{r/(r^2+\omega^2L^2)}{r^2/(r^2+\omega^2L^2)^2 + \left[\omega C_2 - \omega L/(r^2+\omega^2L^2)\right]^2} = 50\Omega$$

From $\mathbb{R}$

$$C_2 = \frac{L}{(r^2+\omega^2L^2)} \pm \frac{\left[\frac{r}{50\Omega} - \frac{r^2}{r^2+\omega^2L^2}\right]^{1/2}}{r^2+\omega^2L^2}$$

Since: $r \ll \omega L$

$$C_2 = \frac{1}{\omega^2 L} \pm \frac{\left(\frac{r}{50\Omega}\right)^{1/2}}{\omega^2 L}$$

$$\therefore \boxed{C_2 = \frac{1 \pm \sqrt{r/50\Omega}}{\omega^2 L}}$$

From $\mathbb{I}$:

$$\frac{\omega C_2 - 1/\omega L}{\left[\omega C_2 - 1/\omega L\right]^2} + \frac{1}{\omega C_1} = 0$$

again: $r \ll \omega L$ trick

$$C_1 = \mp \sqrt{r/50\Omega} \,/\, \omega^2 L$$

Since $C_1 > 0$; substitute C_2 & solve for C_1

$$\boxed{C_1 = \frac{\sqrt{\frac{r}{50\Omega}}}{\omega^2 L} \quad \& \quad C_2 = \frac{1 - \sqrt{r/50\Omega}}{\omega^2 L}}$$

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32) * when the Tx is on both cross diodes are "on" since the voltage is $> 0.5\,V$. At one end of $\lambda/4$ (right before ground) rf is maximum & at the other end it's zero. Essentially there's no voltage drop thru the $\lambda/4$ & so all current & power is delivered to probe circuit.

* when Tx is off, $\lambda/4$ power falls below $0.5V$ & diodes act like open circuit. Now spin induced voltages ($10^{-5}\,V$) travel smoothly to Rx.

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33) i) magnetization is $-M_z$

ii) magnetization is in z-y plane & has an α angle $= \frac{19\pi}{20}$ to z.

33) ii) $I_z \xrightarrow{\left(\frac{19\pi}{20}\right)_x} I_z c\left(\frac{19\pi}{20}\right) - I_y s\left(\frac{19\pi}{20}\right)$

$$= -0.9876 I_z - 0.156 I_y$$

iii) $I_z \xrightarrow{\left(\frac{19\pi}{40}\right)_x} I_z c\left(\frac{19\pi}{40}\right) - I_y s\left(\frac{19\pi}{40}\right) \xrightarrow{\left(\frac{19\pi}{20}\right)_y}$

$$\left[I_z c\left(\frac{19\pi}{20}\right) + I_x s\left(\frac{19\pi}{20}\right)\right] c\left(\frac{19\pi}{40}\right) - I_y s\left(\frac{19\pi}{40}\right) \xrightarrow{\left(\frac{19\pi}{40}\right)_x}$$

$$I_z c^2\left(\frac{19\pi}{40}\right) c\left(\frac{19\pi}{20}\right) - I_y s\left(\frac{19\pi}{40}\right) c\left(\frac{19\pi}{40}\right) c\left(\frac{19\pi}{20}\right)$$

$$+ I_x s\left(\frac{19\pi}{20}\right) c\left(\frac{19\pi}{40}\right) - I_y c\left(\frac{19\pi}{40}\right) s\left(\frac{19\pi}{40}\right) - I_z s\left(\frac{19\pi}{20}\right) s\left(\frac{19\pi}{40}\right)$$

$$= -0.9998 I_z - 0.06095 I_y + 0.0520 I_x$$

Much better I_z inversion vs. ii)