$$\det(A-\lambda I) = \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix}$$

DON'T THROW OUT

$$= \lambda^2 - (a+d)\lambda + \underset{(ad-bc)}{\det A}$$

$$\begin{vmatrix} -w_D-\lambda & w_D \\ w_D & -w_D-\lambda \end{vmatrix} = \lambda^2 + (-2w_D)\lambda + (ad-bc)$$

$$= \lambda^2 - 2w_D\lambda + \left[\cancel{(-w_d)(-w_d) - (w_D)(w_D)}\right] \; 0$$

$$\cancel{\lambda(\lambda - 2w_D) = 0}$$

$$\cancel{\lambda = 0}$$

$$\cancel{\lambda = 2w_D}$$

$$\lambda^2 - (\mathrm{tr} A)\lambda + \det A$$

$$\lambda^2 - [-w_D + (-w_D)]\lambda + 0$$

$$\lambda^2 - [-2w_D]\lambda = 0$$

$$\lambda^2 + 2w_D\lambda = 0$$

$$\lambda(\lambda + 2w_D) = 0$$

$$\boxed{\begin{aligned} \lambda &= 0 \\ \lambda &= -2w_D \end{aligned}}$$

For $\lambda = 0$

$$\begin{pmatrix} -\omega_D-\lambda & \omega_D \\ \omega_D & -\omega_D-\lambda \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$\lambda = 0$

$\lambda = -2\omega_D$

$$(-\omega_D-\lambda)c_1 + \omega_D c_2 = 0$$

$$\omega_D c_1 + (-\omega_D-\lambda)c_2 = 0$$

$$\omega_D c_1 + (-\omega_D)c_2 = 0$$

$$\omega_D(c_1 - c_2) = 0$$

$$c_1 = c_2$$

$$\therefore\ |c_1^2| + |c_2^2| = 1$$

$$c_2^2 + c_2^2 = 1$$

$$2c_2^2 = 1 \qquad c_2 = \frac{1}{\sqrt{2}}$$

$$c_1 = \frac{1}{\sqrt{2}}$$

$$\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$$\Rightarrow \frac{1}{\sqrt{2}}|\alpha\beta\rangle + \frac{1}{\sqrt{2}}|\beta\alpha\rangle$$

$$\frac{1}{\sqrt{2}}\left(|\alpha\beta\rangle + |\beta\alpha\rangle\right)$$

$\lambda = -2w_D$

$$\begin{pmatrix} -w_D - \lambda & w_D \\ w_D & -w_D - \lambda \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$$(-w_D - \lambda) c_1 + w_D c_2 = 0$$
$$w_D c_1 + (-w_D - \lambda) c_2 = 0$$

$$(-w_D - (-2w_D) c_1 + w_D c_2 = 0$$
$$w_D c_1 + (-w_D - (-2w_D) c_2 = 0$$

$$w_D c_1 + w_D c_2 = 0$$
$$w_D c_1 + w_D c_2 = 0$$

$|1\rangle$ $|2\rangle$ $|3\rangle$ $|4\rangle$

$\alpha\alpha$ $\alpha\beta$ $\beta\alpha$ $\beta\beta$

$$w_D (c_1 + c_2) = 0$$
$$c_1 = -c_2$$

$\alpha\alpha$

$\alpha\beta$

$\beta\alpha$

$\beta\beta$

$$|c_1^2| + |c_2^2| = 1$$
$$|(-c_2)^2| + c_2^2 = 1$$
$$c_2^2 = \frac{1}{2}$$

Evals

$c_2 = \frac{1}{\sqrt{2}}$

$\therefore c_1 = \frac{-1}{\sqrt{2}}$

$|$

$|\alpha\beta\rangle$

$|\alpha\alpha\rangle$

$\Rightarrow -\frac{1}{\sqrt{2}} |\alpha\beta\rangle + \frac{1}{\sqrt{2}} |\beta\alpha\rangle$

Levine; p214 1

Find The Eigenvalues

& 2) Normalize eigen vectors OF This hermitian matrix

$$A = \begin{pmatrix} 3 & 2i \\ -2i & 0 \end{pmatrix}$$

$A = A^{T*}$

$$\begin{pmatrix} 3 & 2i \\ -2i & 0 \end{pmatrix} = \begin{pmatrix} 3 & -2(-i) \\ 2(-i) & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 2i \\ -2i & 0 \end{pmatrix}$$ ✓ real values

$$\begin{pmatrix} 3 & 2i \\ -2i & 0 \end{pmatrix} \Rightarrow \begin{vmatrix} 3-\lambda & 2i \\ -2i & -\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(-\lambda) - (-2i)(2i) = 3(-\lambda) - \lambda(-\lambda) - (-2i)(2i)$$

$$= -3\lambda + \lambda^2 - (-4i^2)$$

$$= \underset{A}{\lambda^2} \underset{B}{- 3\lambda} \underset{C}{- 4}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-4)}}{2} = \frac{3 \pm \sqrt{9+16}}{2}$$

$$= \frac{3 \pm \sqrt{25}}{2} = \frac{3 \pm 5}{2}$$

$$= \frac{8}{2}, \frac{-2}{2}$$

$$\lambda = 4, -1$$

2

For root: $\lambda = 4$

$$\begin{vmatrix} 3-\lambda & 2i \\ -2i & -\lambda \end{vmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0$$

$$\Rightarrow (3-\lambda)c_1 + 2i(c_2) = 0$$
$$-2i\cdot(c_1) - \lambda\cdot c_2 = 0$$

$$(3-4)(c_1) + 2i(c_2) = 0 \Rightarrow -c_1 + 2i c_2 = 0$$
$$-2i(c_1) - (4)\cdot c_2 = 0 \qquad -2i c_1 - 4c_2 = 0$$

$$\boxed{c_1 = 2i c_2}$$

use normalization

$$1 = |c_1^2| + |c_2^2|$$
$$1 = |(2i c_2)^2| + |c_2^2|$$
$$1 = 4c_2^2 + c_2^2$$
$$1 = 5c_2^2$$
$$c_2 = \frac{1}{\sqrt{5}}$$

$$1 = |c_1^2| + |c_2^2|$$
$$1 = (2i c_2)^2 + c_2^2$$
$$1 = 4i^2 c_2^2 + c_2^2$$
$$1 = -4c_2^2 + c_2^2$$
$$1 = -3c_2^2$$
$$\sqrt{\frac{-1}{3}} = c_2$$

$$\therefore \boxed{\begin{aligned} c_1 &= 2i c_2 = 2i\frac{1}{\sqrt{5}} \\ c_2 &= \frac{1}{\sqrt{5}} \end{aligned}}$$

For $\lambda = -1$

$c_1^{(2)} = \frac{-i}{\sqrt{5}}$ $c_2^{(2)} = \frac{2}{\sqrt{5}}$

$(3-\lambda)c_1 + 2i \cdot c_2 = 0$

$-2i \cdot c_1 - \lambda c_2 = 0$

$(3--1)c_1 + 2i c_2 = 0$ $\Rightarrow$ $\cancel{4} c_2 = 2i c_1$

$-2i c_1 + \cancel{x} c_2 = 0$ $\quad c_2 = \frac{2i c_1}{\cancel{x}}$

$|c_1^2| + |c_2^2| = 1$ $\quad c_1 = \frac{1}{\sqrt{5}}$

$|c_1^2| + \left|\frac{2i c_1^2}{\cancel{x}}\right| = 1$ $\quad \cancel{c_2 = 2i \cdot \left(\frac{i}{\sqrt{5}}\right) = \frac{2}{\sqrt{5}}}$

$c_1^2 + 4c_2^2 = 1$

$3-(-1)c_1 + 2i c_2 = 0$ $\quad 4c_1 + 2i c_2 = 0$

$-2i c_1 - (-1)c_2 = 0$ $\quad -2i c_1 + c_2 = 0$

$c_1^2 + c_2^2 = 1$

$\vdots$

$c_1 = \frac{-i}{\sqrt{5}}$

$c_2 = \frac{2}{\sqrt{5}}$

$c_2 = 2i c_1$ $\Rightarrow$ $\cancel{4(2ic_1) + 2ic}$

$\cancel{4c_1 + 2i(2ic_1) = 0}$

$\cancel{4c_1 + 4i^2 c_1 = 0}$

$\cancel{|c_1^2| + |c_2^2| = 1}$

$\cancel{c_1^2 + |(2ic_1)^2| = 1}$

$\cancel{c_1^2 + |-4c_1^2| = 1}$

$\cancel{5c_1^2 = 1}$

$\cancel{c_1 = \frac{1}{\sqrt{5}}}$ & $\cancel{c_2 = 2ic_1 = \frac{2i}{\sqrt{5}}}$

$\begin{pmatrix} 4c_1 + 2ic_2 \\ -2ic_1 \quad c_2 \end{pmatrix}$

normalize ev are then

$\vec{c}^{(1)} = \begin{pmatrix} 2i/\sqrt{5} \\ 1/\sqrt{5} \end{pmatrix}$ $\quad \vec{c}^{\,2} = \begin{pmatrix} -i/\sqrt{5} \\ 2/\sqrt{5} \end{pmatrix}$

$\frac{\text{II}}{2} \Rightarrow \boxed{\rho_{mm''}(\tau) = -F_y^{mm''}}$

& after trace $\boxed{A_{mm''} \propto |\langle m|F_+|m''\rangle|^2}$

$$\rho_{mm''} = -F_y^{mm''} = \frac{F_+^{mm''} - F_-^{mm''}}{2i}$$

$$\therefore\ A_{mm''} = \frac{-1}{2i}\left(F_+^{mm''} - F_-^{mm''}\right)\cdot F_+^{m''m}$$

$$= \frac{-1}{2i}\left[\left(F_+^{mm''}\right)\left(F_+^{m''m}\right) - \left(F_-^{mm''}\right)\left(F_+^{m''m}\right)\right]$$

$$I_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$I_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$F_+ = \begin{array}{c|c} m \quad m' \quad m'' \quad m''' & \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{matrix} m \\ m' \\ m'' \\ m''' \end{matrix} \end{array}$$

$$F_- = \begin{array}{ccc} \begin{matrix} m \\ m' \\ m'' \\ m''' \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} & \begin{matrix} m''' \\ m'' \\ m' \\ m \end{matrix} \\ & m''' \quad m'' \quad m' \quad m & \end{array}$$

6) Demonstrate for 2-spins the signal after $\pi/2$ pulse is given by resonances w/ intensity

$$\boxed{A_{mm''} \propto |\langle m|F_+|m''\rangle|^2}$$ — intensity of sigs after $\pi/2$ pulse

$$s(t) = Tr(\rho(t)\cdot F_+)$$

$$\rho_{eq} = 1 + a_z I_z \propto I_z$$

$$\beta = \omega_1 \tau$$

$$\therefore \boxed{\rho(\tau) = e^{+i\omega_1\tau I_x}\, \rho_{eq}\, e^{-i\omega\tau I_x}}$$

← dens. after pulse

not detect

$$\rho(\tau) = a_z[\cancel{c(\beta) I_z} - s(\beta) I_y]$$

$$\rho(\tau) = -a_z s(\beta) I_y$$

$$\rho(\tfrac{\pi}{2}) = -a_z I_y$$

$$s(t) = Tr[\rho(t)\cdot I_+] \qquad I_+ = \begin{pmatrix}0&1\\0&0\end{pmatrix}$$

$$F_+ = \frac{\hbar}{2}\begin{pmatrix}0&1\\0&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} + \frac{\hbar}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}$$

$\hat{I}^{\alpha} = \frac{1}{2}\hat{\mathbb{1}} + I_z$ $\qquad$ $\hat{I}^{\beta} = \frac{1}{2}\mathbb{1} - I_z$

$\hat{I}^{\alpha} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\qquad$ $\hat{I}^{\beta} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$I^{-} = |\beta\rangle\langle\alpha|$

$I^{+} = |\alpha\rangle\langle\beta|$

$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} =$

$= \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}$

$I^{+}|\beta\rangle = |\alpha\rangle\langle\beta|\beta\rangle = |\alpha\rangle$

$I^{-}|\alpha\rangle = |\beta\rangle\langle\alpha|\alpha\rangle = |\beta\rangle$

$$-ppm = \frac{\nu_x - \nu_{tms}}{\nu_{tms}} \cdot 10^6$$

$$E = -\mu \cdot B_0$$
$$= -\gamma I B_0$$

$$\mu_1 = \gamma_1 I_1$$

$$H = H_Z + H_D$$

$$|\alpha_1 \alpha_2\rangle = |1\rangle$$
$$|\alpha_1 \beta_2\rangle = |2\rangle$$
$$|\beta_1 \alpha_2\rangle = |3\rangle$$
$$|\beta_1 \beta_2\rangle = |4\rangle$$

$$H_{11} = \langle 1|H|1\rangle$$
$$= \langle 1|H_Z|1\rangle + \langle 1|H_D|1\rangle$$
$$= \langle \alpha_1\alpha_2|H_Z|\alpha_1\alpha_2\rangle + \langle \alpha_1\alpha_2|H_D|\alpha_1\alpha_2\rangle$$

$$= \langle \alpha_1\alpha_2|-\omega_0 I_{1z}|\alpha_1\alpha_2\rangle$$
$$+ \langle \alpha_1\alpha_2|-\omega_0 I_{2z}|\alpha_1\alpha_2\rangle$$
$$+ \langle \alpha_1\alpha_2|H_D|\alpha_1\alpha_2\rangle$$

$H_Z = -\omega_0(I_{1z} + I_{2z})$

I_{1z} acts on α_1 only

I_{2z} acts " α_2 "

$$= \overbrace{\langle \alpha_1|\alpha_1\rangle}^{1} \langle \alpha_2|-\omega_0 I_{2z}|\alpha_2\rangle$$
$$+ \overbrace{\langle \alpha_2|\alpha_2\rangle}^{1} \langle \alpha_1|-\omega_0 I_{1z}|\alpha_1\rangle$$
$$+ DIP$$

$I_{1z}|\alpha\rangle = \frac{1}{2}|\alpha\rangle$

$$= -\frac{\omega_{02}}{2}\langle \alpha_2|\alpha_2\rangle - \frac{\omega_{01}}{2}\langle \alpha_1|\alpha_1\rangle + DIP$$

Diff $\langle \alpha_1 \alpha_2 | H_0 | \alpha_1 \alpha_2 \rangle = \langle \alpha_1 \alpha_2 | A \ldots F | \alpha_1 \alpha_2 \rangle$

Note: I_+ & I_- Terms $\to 0$ (ie $B - F \to 0$)

$$A = \frac{\omega_0}{2}(P_2) I_{1z} I_{2z} = A' I_{1z} I_{2z}$$

$$\langle \alpha_1 \alpha_2 | I_{1z} I_{2z} | \alpha_1 \alpha_2 \rangle = A' \Big[\cancel{\langle \alpha_1 | \alpha_1 \rangle}^{1} \langle \alpha_2 | I_{2z} | \alpha_2 \rangle + \cancel{\langle \alpha_2 | \alpha_2 \rangle}^{1} \langle \alpha_1 | I_{1z} | \alpha_1 \rangle \Big]$$

$$= A' \cdot \frac{1}{2} \langle \alpha_2 | \alpha_2 \rangle \cdot \langle \alpha_1 | \alpha_1 \rangle \cdot \frac{1}{2}$$

$$\langle | H | \rangle = \frac{A'}{4}$$

$$\text{i)}\quad \boxed{H_{11} = -\frac{\omega_{01}}{2} - \frac{\omega_{02}}{2} + \frac{A'}{4}}$$

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & H_{14} \\ H_{21} & H_{22} & & \\ H_{31} & & H_{33} & \\ H_{41} & & & H_{44} \end{pmatrix}$$

$\Sigma = -\omega_{01} - \omega_{02}$

$\Delta = \omega_{01} - \omega_{02}$

= Chem Shift Diff

Homonuc DIP

$$\hat{H} = \begin{pmatrix} \Sigma+\omega_D & 0 & 0 & 0 \\ 0 & \Delta-\omega_D & \omega_D & 0 \\ 0 & \omega_D & -\Delta-\omega_D & 0 \\ 0 & 0 & 0 & -\Sigma+\omega_D \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\Delta = \omega_{01} - \omega_{02}$

$\Sigma = \frac{-\omega_{01}}{2} - \frac{\omega_{02}}{2}$

$\cancel{\Delta = \frac{\omega_{02} - \omega_{01}}{2}}$

Eigenstates	Eigen Values	E levels
$\lvert\beta\beta\rangle$	$\cancel{\Sigma+\omega_D}$ $\Sigma+\omega_D$	— $\lvert 4\rangle$
$\frac{\omega_D}{(\omega_D^2+2\Delta^2)^{1/2}}\lvert\alpha\beta\rangle + \left(\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}\right)^{1/2}\lvert\beta\alpha\rangle$	$-\omega_D + (\omega_D^2+\Delta^2)^{1/2}$	— $\lvert 3\rangle$
$\frac{\omega_D}{(\omega_D^2+2\Delta^2)^{1/2}}\lvert\alpha\beta\rangle - \left(\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}\right)^{1/2}\lvert\beta\alpha\rangle$	$-\omega_D - (\omega_D^2+\Delta^2)^{1/2}$	— $\lvert 2\rangle$
$\lvert\alpha\alpha\rangle$	$-\Sigma+\omega_D$	— $\lvert 1\rangle$

Observable transitions

$\cancel{\rho_{42}}$ ρ_{21}

ρ_{31}

ρ_{42}

ρ_{43}

TRANSITION INTENSITIES

$$A_{12} = |\langle 2|F_+|1\rangle|^2$$

$$= \left(0 \quad \frac{\omega_D}{\sqrt{\omega_D^2+2\Delta^2}} \quad -\sqrt{\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}} \quad 0\right)\cdot\begin{pmatrix}0&1&1&0\\0&0&0&1\\0&0&0&1\\0&0&0&0\end{pmatrix}\cdot\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$A_{12} = \left(\frac{\omega_D}{\sqrt{\omega_D^2+2\Delta^2}} - \sqrt{\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}}\right)^2$$

$$A_{13} = \left(\frac{\omega_D}{\sqrt{\omega_D^2+2\Delta^2}} + \sqrt{\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}}\right)^2$$

$$A_{24} = \left(\frac{\omega_D}{\sqrt{\omega_D^2+2\Delta^2}} - \sqrt{\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}}\right)^2$$

$$A_{34} = \left(\frac{\omega_D}{\sqrt{\omega_D^2+2\Delta^2}} + \sqrt{\frac{\Delta^2+\omega_D^2}{2\Delta^2+\omega_D^2}}\right)^2$$

J-coupled Hamiltonians

$$\boxed{H = -\sum_k \omega_k I_{zk} + \sum_{i<k}\sum_k J_{ik} \vec{I}_i \cdot \vec{I}_k}$$

1) heteronuc $|\omega_i - \omega_k| \gg J_{ik}$

$$\boxed{H = -\omega_1 I_{1z} - \omega_2 I_{2z} + J_{12} I_{1z} I_{2z}}$$

2) Homonuc: $|\omega_i - \omega_k| \approx J$

$$H = -\omega_1 I_{1z} - \omega_2 I_{2z} + J \vec{I}_1 I_2$$

$$\boxed{H = -\omega_1 I_{1z} - \omega_2 I_{2z} + J\left[I_{1z}I_{2z} + \frac{I_{1+}I_{2-} + I_{1-}I_{2+}}{2}\right]}$$

Homonuc

Eigen Vectors

$|\beta\beta\rangle = |4\rangle$

$c|\alpha\beta\rangle + d|\beta\alpha\rangle = |3\rangle$

$a|\alpha\beta\rangle + b|\beta\alpha\rangle = |2\rangle$

$|\alpha\alpha\rangle$

$a = c\left(\frac{\theta}{2}\right)$

$b = s\left(\frac{\theta}{2}\right)$

$c = -s\left(\frac{\theta}{2}\right)$

$d = c\left(\frac{\theta}{2}\right)$

Eigen Values

$\frac{\Sigma}{2} + \frac{J}{4}$	$\lvert 4\rangle$
$-\frac{\gamma B_{ef}}{2} - \frac{J}{4}$	$\lvert 3\rangle$
$+\frac{\gamma B_{ef}}{2} - \frac{J}{4}$	$\lvert 2\rangle$
$-\frac{\Sigma}{2} + \frac{J}{4}$	$\lvert 1\rangle$

Note:

$$\Delta\omega_1 = \omega_1 - \omega$$

$$\Delta\omega_2 = \omega_2 - \omega$$

$$\boxed{\begin{aligned} \Sigma &= \Delta\omega_1 + \Delta\omega_2 \\ \Delta &= \Delta\omega_1 - \Delta\omega_2 \end{aligned}}$$

$E_{|4\rangle} \quad \frac{\Sigma}{2} + \frac{J}{4}$

$E_{|3\rangle} \quad \frac{-\gamma B_{eff}}{2} - \frac{J}{4}$

$E_{|2\rangle} \quad \frac{\gamma B_{eff}}{2} - \frac{J}{4}$

$E_{|1\rangle} \quad -\frac{\Sigma}{2} + \frac{J}{4}$

calc Frequencies

$$\nu_{ij} = \frac{E_i - E_j}{\hbar}$$

Intensities

$$I_{ij} \propto \left(\langle i | F_+ | j \rangle\right)^2$$

Homonuc Dip

$$
\begin{array}{ccccc}
 & \alpha\alpha & \alpha\beta & \beta\alpha & \beta\beta \\
|\alpha\alpha & \Sigma + \omega_D & & & \\
\alpha\beta & & \Delta - \omega_D - \lambda & \omega_D & \\
\beta\alpha & & \omega_D & -\Delta - \omega_D - \lambda & \\
\beta\beta & & & & -\Sigma + \omega_D - \lambda
\end{array}
$$

$\boxed{\Delta = \omega_{01} - \omega_{02} \qquad \Sigma = \frac{-\omega_{01}}{2} - \frac{\omega_{02}}{2}}$

Identical ~~EQ~~ Spins

$\Delta = 0$

$\Sigma = -\omega_0$

$\lambda = 0 \; [c_1 = c_2 = \frac{1}{\sqrt{2}}$

$\lambda = 2\omega_D \; [c_1 = -\frac{1}{\sqrt{2}} \quad c_2 = \frac{1}{\sqrt{2}}$

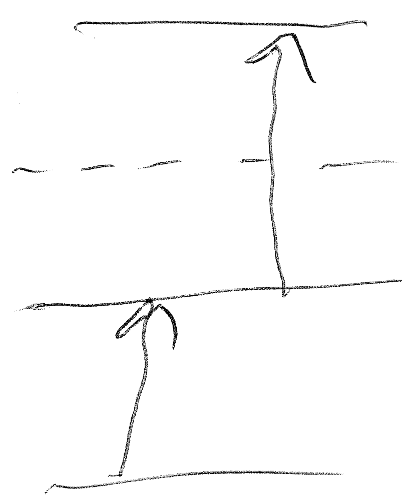
$|4\rangle = |\beta\beta\rangle$

$|3\rangle = a|\alpha\beta\rangle + b|\beta\alpha\rangle$

$|2\rangle = c|\alpha\beta\rangle + d|\beta\alpha\rangle$

$|1\rangle = |\alpha\alpha\rangle$

Eig States	Eig Vals
$\vert 4\rangle = \vert\beta\beta\rangle$	$\omega_0 + \omega_D$
$\vert 3\rangle = \frac{1}{\sqrt{2}}\vert\alpha\beta\rangle + \frac{1}{\sqrt{2}}\vert\beta\alpha\rangle$	0
$\vert 2\rangle = \frac{1}{\sqrt{2}}\vert\beta\alpha\rangle - \frac{1}{\sqrt{2}}\vert\alpha\beta\rangle$	$-2\omega_D$
$\vert 1\rangle = \vert\alpha\alpha\rangle$	$-\omega_0 + \omega_D$



J-coup Hamilt Transition Freqs

$$A_{12} = (\langle 2|F_+|1\rangle)^2 = \begin{pmatrix} 0 & a & b & 0 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A_{12} = (\ \ ) \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = (a+b)^2 = c^2\left(\tfrac{\theta}{2}\right) + s^2\left(\tfrac{\theta}{2}\right) + 2c\left(\tfrac{\theta}{2}\right)s\left(\tfrac{\theta}{2}\right)$$

$$A_{13} = \langle 3|F_+|1\rangle = \begin{pmatrix} 0 & c & d & 0 \end{pmatrix} \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= (\ \ ) \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} = (c+d)^2 = \left[-s\left(\tfrac{\theta}{2}\right) + c\left(\tfrac{\theta}{2}\right)\right]^2$$

$$\boxed{A_{13} = c^2\left(\tfrac{\theta}{2}\right) - 2c\left(\tfrac{\theta}{2}\right)s\tfrac{\theta}{2} + s^2\left(\tfrac{\theta}{2}\right)}$$

$$A_{24} = \langle 4|F_+|2\rangle = (1\,0\,0\,0) \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ b \\ a \\ 0 \end{pmatrix}$$

$$= (1\,0\,0\,0) \begin{pmatrix} a+b \\ 0 \\ 0 \\ 0 \end{pmatrix} = (a+b)^2 = A_{12}$$

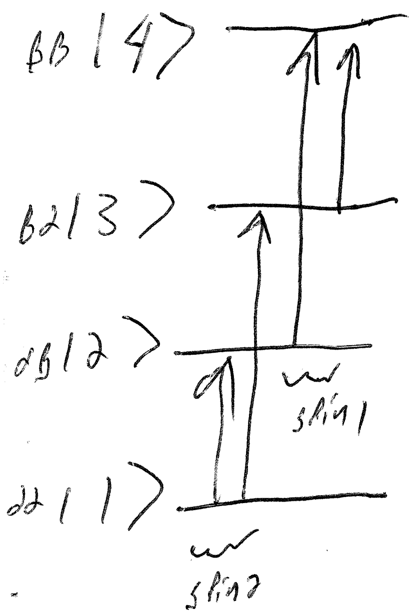
$$A_{34} = \langle 4|F_+|3\rangle = (1\,0\,0\,0) \begin{pmatrix} 0&1&1&0 \\ 0&0&0&1 \\ 0&0&0&1 \\ 0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ c \\ d \\ 0 \end{pmatrix}$$

$$= (\ \ ) \begin{pmatrix} c+d \\ 0 \\ 0 \\ 0 \end{pmatrix} = (c+d)^2 = A_{13}$$

$$\boxed{\begin{aligned} A_{12} = A_{24} &= (a+b)^2 = \left(c\left(\tfrac{\theta}{2}\right) + s\left(\tfrac{\theta}{2}\right)\right)^2 \\ A_{13} = A_{34} &= (c+d)^2 = \left[c\left(\tfrac{\theta}{2}\right) - s\left(\tfrac{\theta}{2}\right)\right]^2 \end{aligned}}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{matrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{matrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = F_+$$

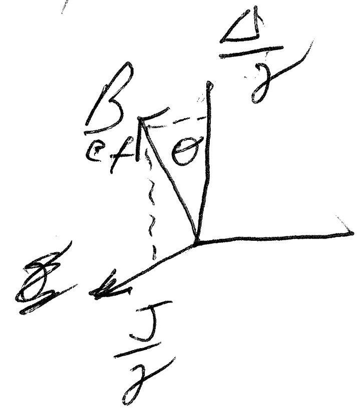


$\frac{\Sigma}{2} + \frac{J}{4}$

$-\frac{\gamma B_{ef}}{2} - \frac{J}{4}$

$\frac{\gamma B_{ef}}{2} - \frac{J}{4}$

$-\frac{\Sigma}{2} + \frac{J}{4}$



$$\gamma B_{ef} = \sqrt{\Delta^2 + J^2}$$

The Frequencies

$$\Omega_{21} = \frac{\gamma B}{2} - \frac{J}{4} - \left(-\frac{\Sigma}{2} + \frac{J}{4}\right) = \frac{\gamma B_{ef}}{2} - \frac{J}{2} + \frac{\Sigma}{2}$$

$$\Omega_{31} = \frac{-\gamma B}{2} - \frac{J}{4} - \left(-\frac{\Sigma}{2} + \frac{J}{4}\right) = \frac{-\gamma B}{2} - \frac{J}{2} + \frac{\Sigma}{2}$$

$$\Omega_{43} = \frac{\Sigma}{2} + \frac{J}{4} - \left(\frac{-\gamma B}{2} - \frac{J}{4}\right) = \frac{\gamma B}{2} + \frac{J}{2} + \frac{\Sigma}{2}$$

$$\Omega_{42} = \frac{\Sigma}{2} + \frac{J}{4} - \left(\frac{+\gamma B}{2} - \frac{J}{4}\right) = \frac{-\gamma B}{2} + \frac{J}{2} + \frac{\Sigma}{2}$$

$$S(t) = \sum_{r=1}^{4} \sum_{s=1}^{4} A_{rs} \cdot \exp(i\Omega_{rs} - \lambda) \cdot t$$

① Supp. FFT of Typical Transfer Funcs

* illustrating behavior of
  cos FFT & sin FFT on Trig Funcs

* cos = even ∴ c·c = even   keep in mind
  sin = odd   c s = odd

* want absorptive not dispersive pks ∴

  cos FFT (even) ⇒ Abs

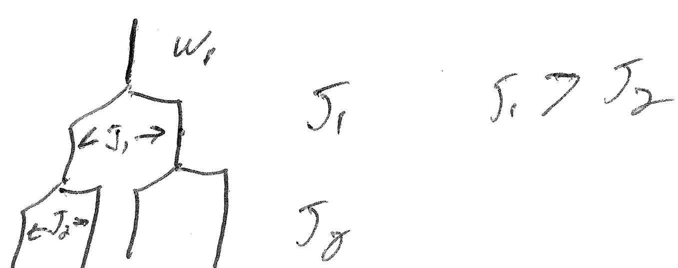
  cos FFT (odd) ⇒ Disp   etc...

---

* ~~Note trig identities on following~~ page

Want to write Tx funcs w/ inc J-values like:

$S(\omega) \cdot C(J_1) \cdot C(J_2)$ so triangle goes



It is best to write out like this too:

$S(w_1) \cdot C(J_1) \cdot C(J_2)$

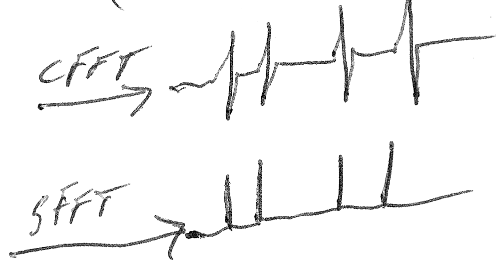
$(Sw \cdot C_J) \cdot C(J_2)$

$= (S + S) \cdot C(J_2)$

$= (S \cdot C_J) + (S \cdot C)$

$= (S + S) + (S + S)$

$(S+S) + (S+S)$



② OE

$cw_1 \cdot sJ_1 \cdot sJ_2$

$(cw_1 \cdot sJ_1) \cdot sJ_2$

$= (s + s) \cdot s$

$= (s \cdot s) + (s \cdot s)$

$= (c - c) + (c - c)$

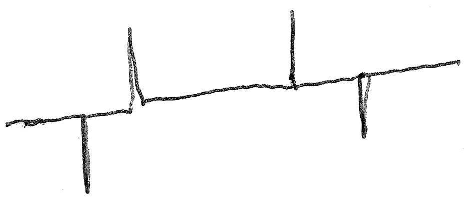
$c \cdot s = \dfrac{s(\alpha+\beta) - s(\alpha-\beta)}{2}$

order matters

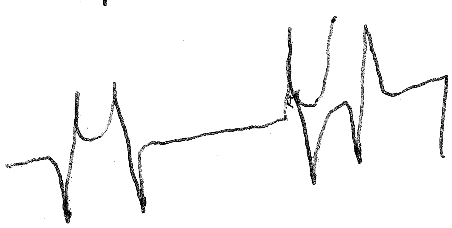
$s \cdot c = \dfrac{s(\alpha+\beta) + s(\alpha-\beta)}{2}$

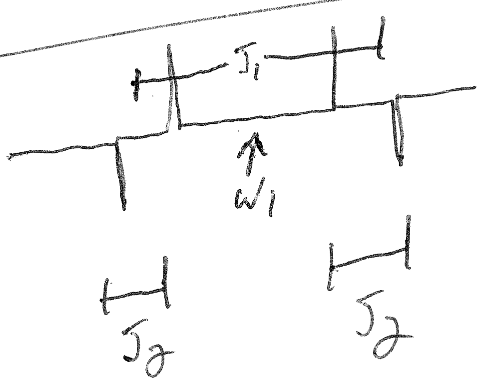
$s \cdot s = \dfrac{c(\alpha-\beta) - c(\alpha+\beta)}{2}$

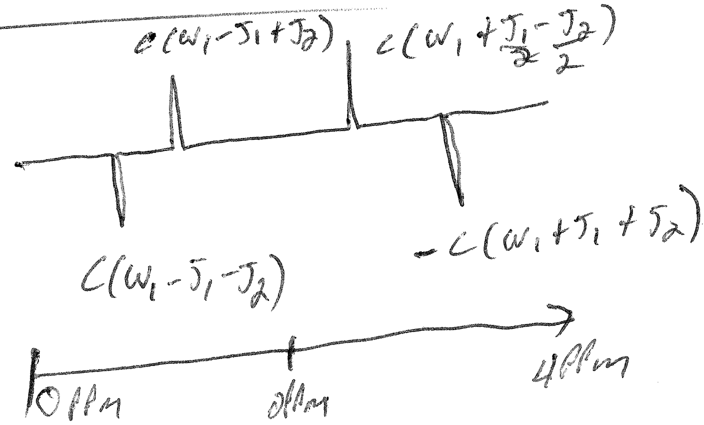
cFFT ⟹

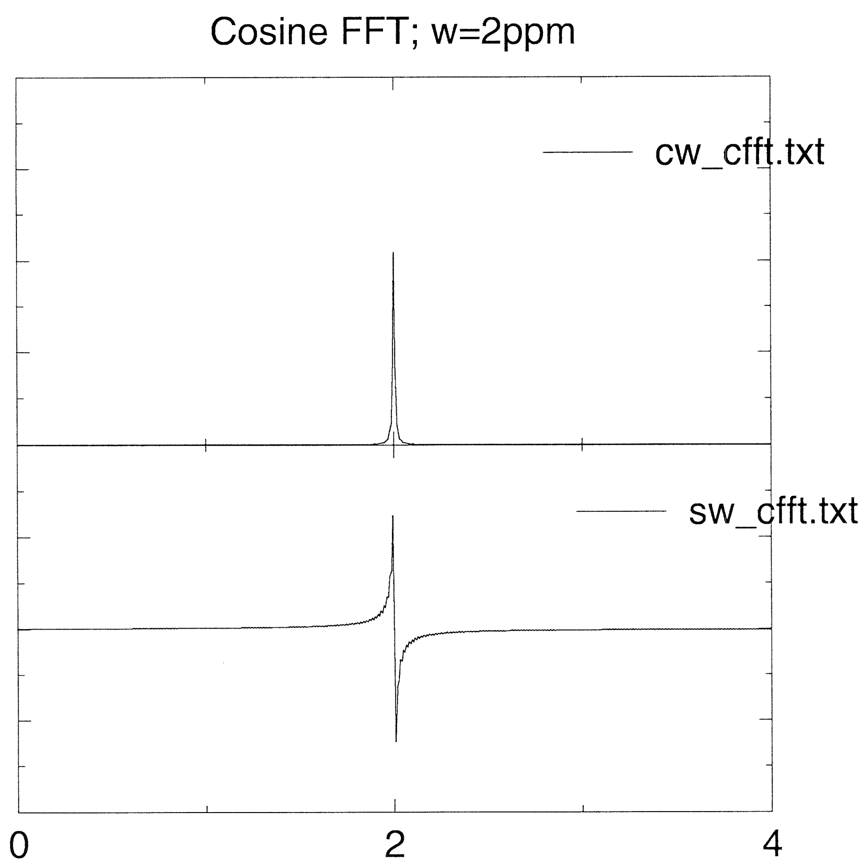


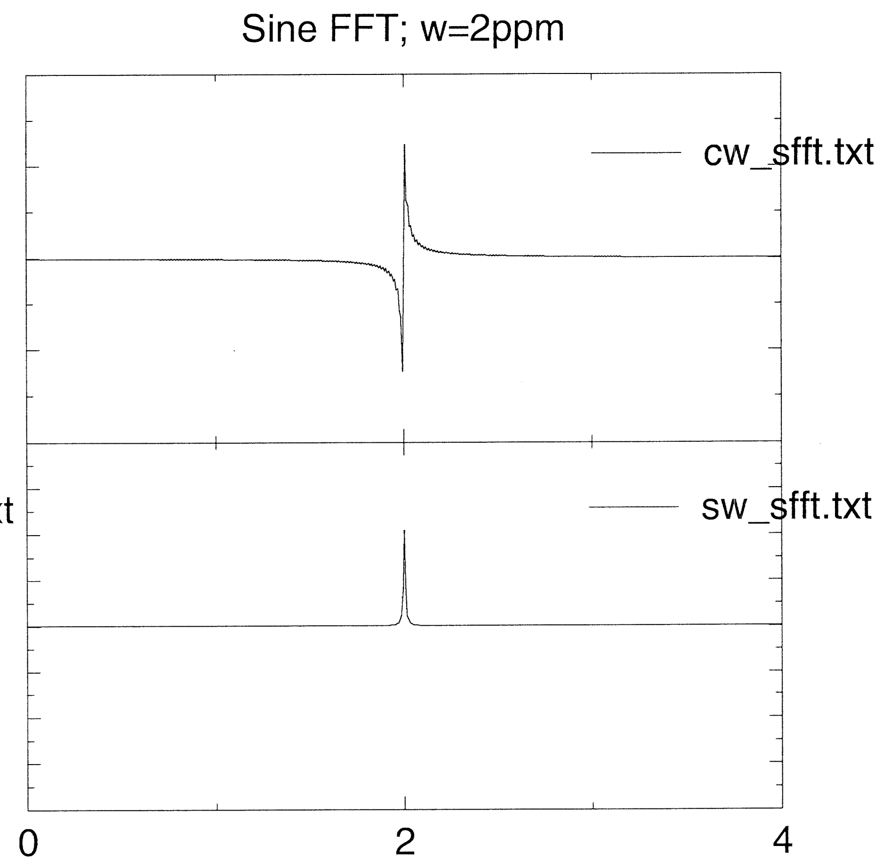
sFFT ⟹

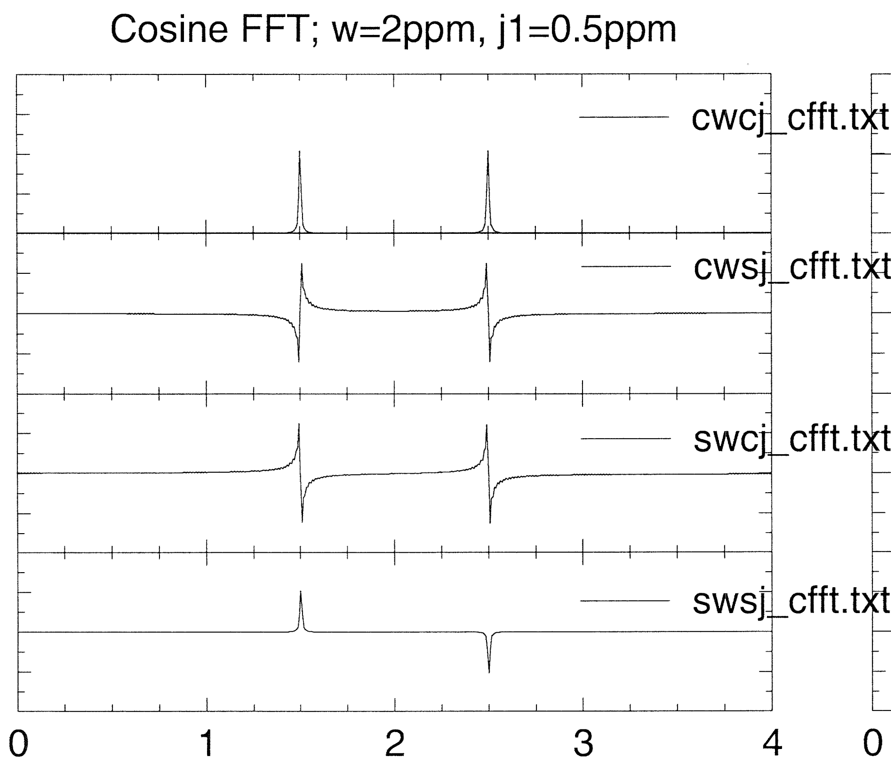


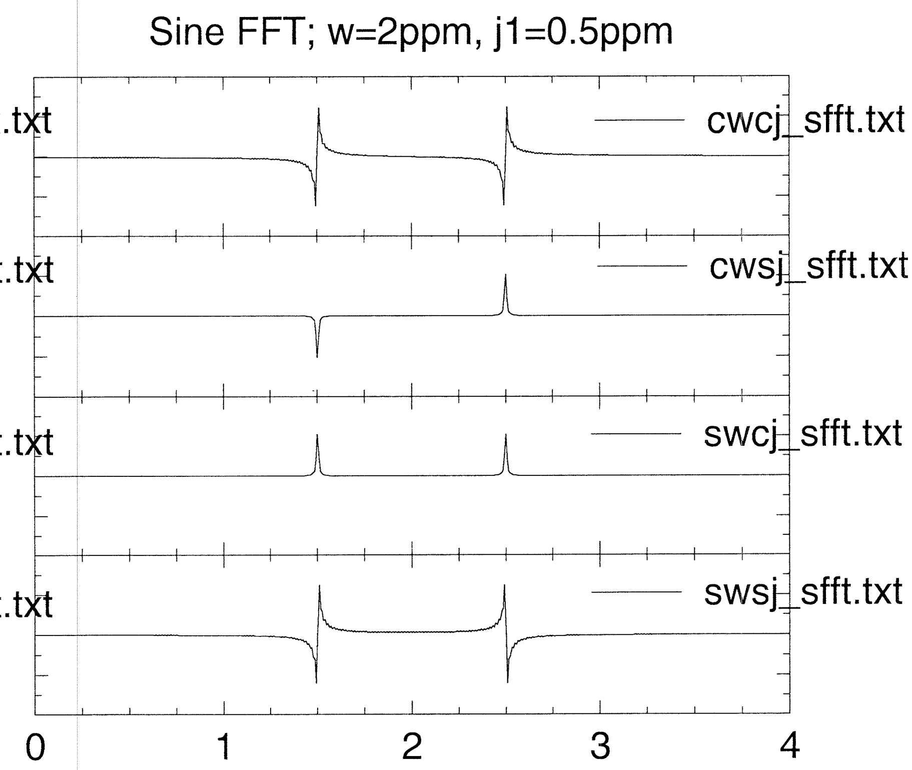





Cosine FFT; w=2ppm
cw_cfft.txt
sw_cfft.txt
0
2
4


Sine FFT; w=2ppm
cw_sfft.txt
sw_sfft.txt
0
2
4





TRIG IDENTITIES

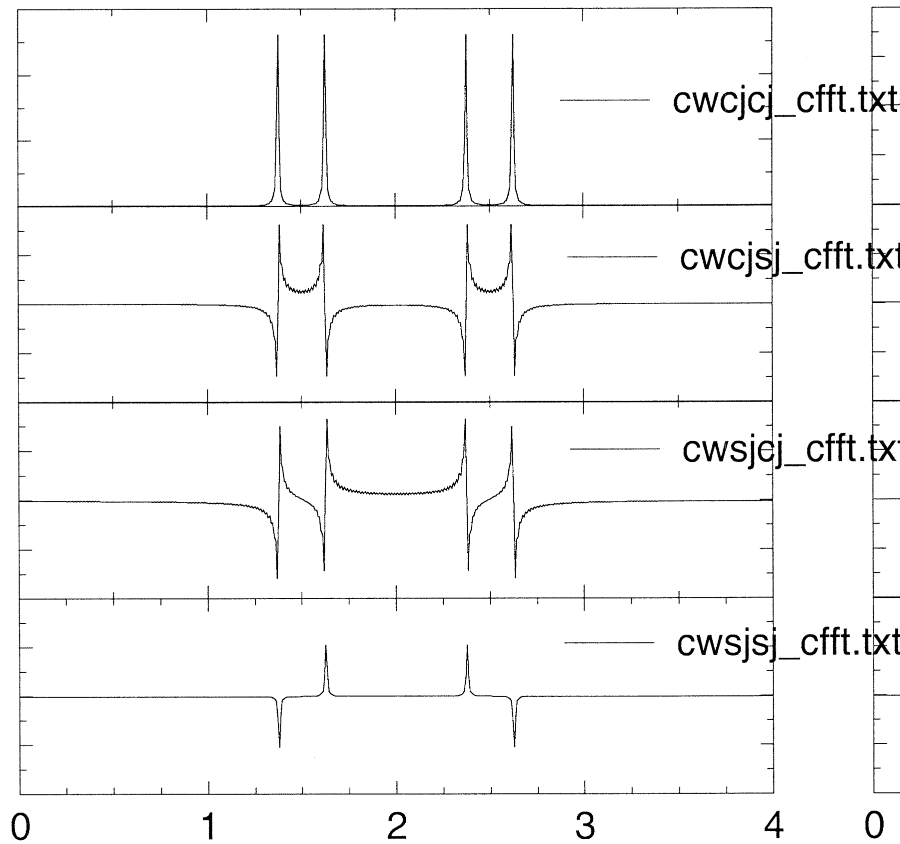
$$C(\alpha)C(\beta) = \frac{C(\alpha+\beta) + C(\alpha-\beta)}{2}$$

$$S(\alpha)C(\beta) = \frac{S(\alpha+\beta) + S(\alpha-\beta)}{2} \qquad C(\alpha)S(\beta) = \frac{S(\alpha+\beta) - S(\alpha-\beta)}{2}$$

ORDER MATTERS HERE FOR US!

$$S(\alpha)S(\beta) = \frac{C(\alpha-\beta) - C(\alpha+\beta)}{2}$$

Cosine FFT; w1=2ppm, j1=0.5ppm; j2=0.125ppm



Cosine FFT; w1=2ppm, j1=0.5ppm, j2=0.125ppm